Semester Report

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Introduction:

For two different directions:

1. As we know, the $N \times T$ covariance matrix in the risk function of no-short optimization has a block diagonal structure. Commonly, we assume assume that the elements outside the diagonal identity are all equal to ρ, where $-1/N < \rho \leq 1$. With the ratio $r = N/T$, it is clearly that the critical boundary of zero risk r_c, where the ratio of zero risk = 50%, is equal to 2 for $\rho=0$. And from our previous work, the value of r_c will continue to increase as ρ becomes larger in $(0,1)$. Theoretically the peak value of proportion of zero weights n_0 and zero risk boundary coincide with same value of r. But unpredicted data come out around r_c by the numerical calculation so that we can not get the real r_c. To dig out the "true" n_0, those weights which lead to risk $=0$ are dropped away. Then we can get n_c, the "true" value of n_0 at the critical boundary r_c, which is also a function of ρ.

2. To gain information on heavy quarkonium properties such as thermal modifications of bound states or transport coefficients, studies of the hadronic spectral function ρ in the hot medium are needed. In Lattice QCD, one can calculate is the related Euclidean correlation function as vanishing momentum. To extract the spectral function from the correlation function one needs to solve an ill-posed inverse problem. However, we tried it by a network approach based on Gaussian Mixture Model(GMM), and our results of single-peak model highly match the data from Maximum Entropy Method(MEM). Further more, we built a generalized multi-peak model for test because nobody knows if the real spectral function has only a peak or not. By out experiments, this multi-peak model can only plot a function with a single peak, which proves that the spectral function is a one-peak function.

Summary of research work carried out in in the previous three semesters:

For two different directions:

1. Portfolio Optimization. Although I learnt the replica method in optimization at the very beginning, most of the works have been done are via R language numerically. By playing around no-short optimization risk function, we focus on the $N \times T$ diagonal covariance matrix. The elements ρ outside the diagonal identity in covariance matrix are highly related to the value of zero risk bound r_c, ratio of zero weight n_0 and n_c, the "true" peak of n_0 at r_c. To study the relation between r_c and N, a virtual sample size N_f is introduced as a ruler to

measure real size N and a virtual covariance matrix element ρ_0 is built as $\rho = \rho_0 N/N_f$. With this scale, $N_f \leq N$ keeps the constraint of $0 \leq \rho \leq 1$ from virtual one $0 \leq \rho_0 \leq 1$. After that a general relation between ρ_0 and r_c can be addressed $r_c \approx 0.29 \times N_f \rho_0 + 2$. For n_0, there are two types of relations, for r far away of r_c, the relation looks like $n_0 \approx 0.21 \times ln(r) + constant$; for r close to r_c, it becomes $n_0 \approx ar + b$, where a and b are small constants determined by N and ρ. Of course, in an extremely case $-1/N \leq \rho < 0$, we also find a law between r_c and ρ_0: $r_c \approx 0.75 \times \rho_0$.

2. Spectral Function Reconstruction in Lattice QCD. The work has been done in Python with the package "TensorFlow". To reconstruct the spectral function ρ from correlator G, we start from the Bayes' theorem $P(\rho|z) = P(\rho,z|G)/P(z,\rho|G)$, where z is a Intermediate variables. With the help of Maximum Likelihood Estimate(MLE), we can built a target loss function by this probability relation. Here we assume ρ is directly generated by the hidden variable z but not the noisy correlator G', and then a relation based on The Gaussian Mixture Model(GMM) can be used to construct a network system as expressions of those probabilities of ρ, z, G and G'. By the data from the experiments of Lattice QCD and Variation Autoencoder(VAE) process, this network can be trained well as known functions of probabilities of ρ, z, G and G', which can be used to reconstruct ρ from G'. Then we built a single-peak model at below Tc and above Tc for training and reconstruction. The results of this model are similar to the famous MEM which indicates that our network approach is an effective way to reconstruct the spectral function.

Description of research work carried out in current semester:

1. As a function of $r = N/T$, the peak value of n_0 and zero risk boundary coincide with same r in theory but they are different in the result from numerical calculation via program. To determine n_c, the "real" value of n_0 at $r_ç$, we focus on the value of risk for each weight. By collecting the weights of non-zero risk, the ratio of zero weights is rebuild with "true" data around the critical boundary r_c. Then we can get the real n_c and find it also can be expressed as a function of ρ: $(n_c)^2 = a\ ln(\rho) + b$, is "true" n_0, but the behavior between n_0 and ρ is still on working. $(n_c)^2 = a\ ln(\rho) + b$, where a and b depends on the sample size N. Similarly, we use the rescale function $\rho = \rho_0 N_f/N$, For $N_f = 100$ and any $N \geq 100$, it is clearly that $(n_c)^2 = 0.138765 \times ln(\rho_0) + 1.005$. For $N_f = 150$ and any $N \geq 150$, it becomes $(n_c)^2 = 0.1487657 \times ln(\rho_0) + 1.091$. And for $N_f = 200$ and any $N \geq 150$, the function becomes $(n_c)^2 = 0.1352769 \times ln(\rho_0) + + 1.085$. Because a and b varies in a small range, it is difficult to try out how it relates to N. But I am still working on it. In previous work, we usually set all the elements outside the diagonal identity are all equal to a number ρ. Now imaging those elements as a random value in $(-1/N, 1)$ and keeping the covariance matrix positive-definite, the relation s among r, ρ, r_c and n_c might be more complex. The model will be played around as a further work to find more universal laws.

2. For the reconstruction of spectral function, we improved and optimized the structure and parameters of the network. Then from the first single-peak model, two new models of the spectral function are built to test the network: double-peak and multi-peak models. They are more universally than the single-peak model because it is not clear that how many peaks the spectral function really has. By testing the double-peak model, we find it is hard to plot out a second peak of the spectral function by different sets of training data. Similarly, for the multi-peak model, there is also only a main peak for reconstruction. From those results, we might say that the spectral function in lattice QCD has only a single peak and the location of that peak can be predicted in a range. Our multi-peak model still highly matches the result of MEM, which shows the network is a good method to extract the spectral function. Currently we find that our MLE may have a relation with the MEM via Shannon-Jaynes entropy, this theoretical bridge is under construction now with test. And some new data at above Tc will come out from the other lattice QCD group, I think we should not wait longer for testing our models with that data.

Publication:

A Network Approach To Spectral Function Reconstruction, S-Y. Chen, F-Y. Liu, H-T. Ding, G. Papp, C-B. Yang. (Manuscript finished, checked by the group members.)

Studies in current semester:

ELTE courses:

FIZ/3/059E Evolutionary game theory

FIZ/3/025E Trapped atomic systems

Conferences in current semester:

Online Forum of Lattice QCD group at Central China Normal University

Online discussion with DeepRTP group

Awards:

Stipendium Hungaricum Scholarship

Research Allowance of CCNU For PhD, 2020.02 - 2020.06

Research Allowance of Institute of Particle Physics of CCNU For PhD, For Spring Semester