Semester report

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Ph.D. Thesis title: Investigation of topological superconductors in the presence of interactions

1 Introduction

Isolated Majorana bound states provide robust quantum information storage, but the number of possible topologically protected quantum operators is very limited, thus making them less effective for quantum information processing. A promising alternative is to use parafermions, the generalization of Majorana fermions. Generally speaking, braiding of parafermionic zero modes leads to a richer set of possible unitary transformations acting on the degenerate ground state manifold compared to what is possible with Majoranas, but it requires the presence of strong interaction. The study of these interaction-enabled phases has focused either on microscopic models derived from prototypical clock models, resulting in interaction and superconducting terms that are hard to reproduce experimentally in the Hamiltonian, or on an effective field theoretical description employing a bosonized framework [1–3].

The overarching theme of my research plan is to investigate microscopic models of topological superconductivity in the presence of interactions, focusing on systems with explicit experimental relevance, hosting topologically protected zero energy modes.

I envisaged two objectives for my research plan:

- I: Zero modes at the interface of topological insulators and superconductors
- II: Nanowires for interaction enhanced topological qubits

In this first semester, I focused my effort on Objective I.

2 Research work carried out in current semester

2.1 Model and numerical background

My goal is to create a microscopic model capable of describing the edge states of topological insulators in the presence of superconductivity and interaction, without the massive bulk. One of the simplest ways to do it is to start with a ladder-like lattice model (with $2 \times N$ sites for some $N \in \mathbb{N}$) which is, in the low-energy limit, equivalent to the Hamiltonian of the edge states of a topological insulator. Because of this, I took the kinetic Hamiltonian

$$\begin{aligned} H_{\mathrm{kin}} = &\sum_{n,\sigma} \begin{bmatrix} c^{\dagger}_{n,L,\sigma} & c^{\dagger}_{n,R,\sigma} \end{bmatrix} \begin{bmatrix} -\mu & t \\ t & -\mu \end{bmatrix} \begin{bmatrix} c_{n,L,\sigma} \\ c_{n,R,\sigma} \end{bmatrix} \\ &- \frac{t}{2} \sum_{n,\sigma} \left(\begin{bmatrix} c^{\dagger}_{n+1,L,\sigma} & c^{\dagger}_{n+1,R,\sigma} \end{bmatrix} \begin{bmatrix} i\sigma & 1 \\ 1 & -i\sigma \end{bmatrix} \begin{bmatrix} c_{n,L,\sigma} \\ c_{n,R,\sigma} \end{bmatrix} + \mathrm{h.c.} \right), \end{aligned} \tag{1}$$

where t is a real parameter, μ us the chemical potential, $n \in \mathrm{N}$ is the site degree of freedom, $\zeta \in \{\mathrm{L}, \mathrm{R}\}$ is the newly introduced "side" degree of freedom (that is, our model can be imagined as a ladder), and σ is the spin (with $\uparrow = 1$, $\downarrow = -1$ convention).

The s-wave superconductivity is taken into account via Cooper-pair creation and annihilation, that is

$$H_{\mathrm{sc}} = \sum_{n,\zeta} \Delta_{n,\zeta} \left[c^{\dagger}_{n,\zeta,\uparrow} c^{\dagger}_{n,\zeta,\downarrow} + \mathrm{h.c.} \right]. \tag{2}$$

There are two interaction terms I used; They can be written as

$$H^{(1)}_{\mathrm{int}} = \sum_{n,\zeta} V^{(1)}_{n,\zeta} \left[c^{\dagger}_{n,\zeta,\uparrow} c_{n,\zeta,\downarrow} c^{\dagger}_{n+1,\zeta,\downarrow} c_{n+1,\zeta,\uparrow} + \mathrm{h.c.} \right], \tag{3}$$

$$H_{\rm int}^{(2)} = \sum_{n,\zeta} V_{n,\zeta}^{(2)} \left[c_{n,\zeta,\uparrow}^{\dagger} c_{n,\zeta,\downarrow} c_{n+1,\zeta,\uparrow}^{\dagger} c_{n+1,\zeta,\downarrow} + \text{h.c.} \right]. \tag{4}$$

Both of these interactions conserve time-reversal symmetry. The second interaction $H_{\rm int}^{(2)}$ was suggested in [4], and $H_{\rm int}^{(1)}$ is a simplified version of it. I started to work with $H_{\rm int}^{(1)}$ first, because it conserves the total spin z component, making the calculations faster.

Considering a model with explicit superconductivity and complicated interactions, the only possible Abelian quantum numbers are the spin projection and the particle parity. Even though the limited number of quantum numbers foreshadow challenges in the accurate numerical treatment of the problem, I found that the substantial gap induced by the interaction makes the simulations rather manageable even for ladders with reasonable size.

In the numerical calculations, I was using Python (for exact diagonalizations of non-interacting systems), and the ITensor program's [5] Density Matrix Renormalization Group (DMRG) algorithm [6] implemented in Julia language.

2.2 Identifying the phase diagram

The phase diagram is scanned on a system with length $N = 40$, with one leg of the ladder having interaction in the middle (with length $N_2 = 20$) and superconductivity on its ends (with lengths $N_1 = N_3 = 10$), and the other having superconductivity only (with, of course, both sides having the kinetic term present). These calculations were relatively fast (each parameter point took about 1-2 hours and they were able to run parallel), as it only needed the ground state. A sketch of this configuration can be seen in Figure 1. Phase transitions were detected by studying the maximum matrix dimension of the DMRG calculations (maxlinkdim in the figures) which is, similarly to entropic quantities, strongly related to the correlation lengths.

The phase diagram of the interactions $H_{\rm int}^{(1)}$ and $H_{\rm int}^{(2)}$ can be seen in Figure 1, with $U^{(1)}$ and $U^{(2)}$ denoting the appropriate interaction strength. This system has 4 different phases: the weak interaction phase, where the system is metallic (denoted by W); the strong interaction phase (denoted by S), where the system is gapped and there is no degenerate ground state; and the two separate fourfold degenerate regimes (denoted by $4\times^{(1)}$ and $4\times^{(2)}$ for the appropriate interaction).

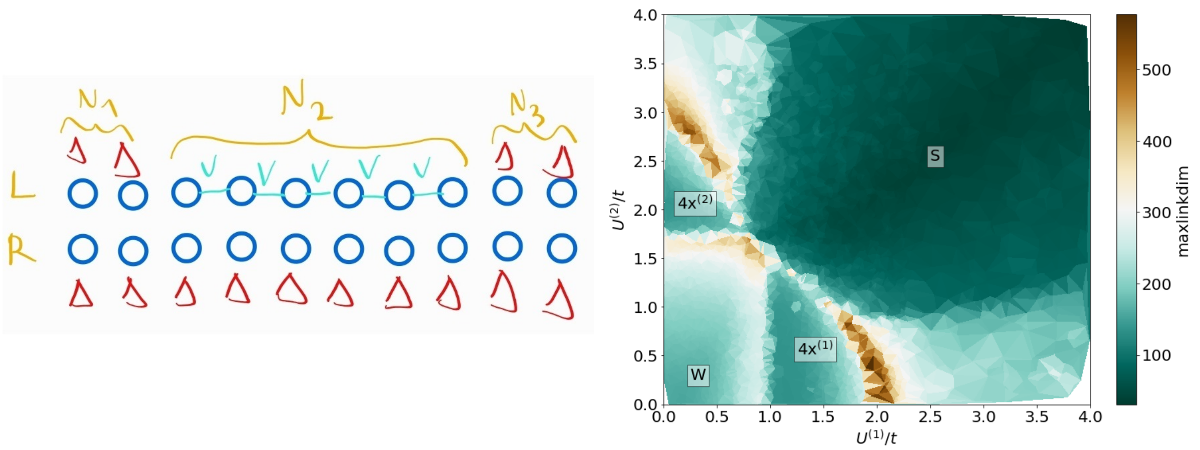

Figure 1: The system used for the phase diagram calculations (left), and phase diagram of the two interactions (right).

2.3 Characterization of the degenerate phase

As can be seen in the previous section, four-fold degenerate ground states can be identified for intermediate couplings for both kind of interactions. Furthermore, I found that these degenerate states are localized at the domain walls. Even though, this high degeneracy does indicate the presence of localized zero energy modes they are not necessarily of parafermionic nature. In fact, their conclusive description can be given by an analysis based on the periodicity of the Josephson spectrum.

The Josephson effect is a current that might be observed between two superconductors separated by a thin insulating barrier at zero voltage. The magnitude of the current is tuned by the phase difference of superconducting segments. Correspondingly, I studied the characteristic phase of model to identify the nature of the observed four-fold groundstate, i.e., for Majorana zero modes and parafermionic states 4π and 8π periodicity is expected [7, 8].

Accordingly, I studied the Josephson spectrum of several different systems, looking for the wished parafermions. The result of a calculation using a system with 4 domain walls with interaction $H_{\text{int}}^{(2)}$ (due to parity conservation, only the even case is shown) is shown in the right of Figure 2, while a sketch of the system is shown in the left. The right leg of the system is characterized by superconductivity, and the left leg has superconductivity (length N_1), interaction (length N_2), superconductivity with phase φ (length N_3), interaction (length N_4) and finally superconductivity again (length N_5). The parameters of the system are $\Delta = 1$, $(N_1, N_2, N_3, N_4, N_5) = (20, 8, 20, 16, 16)$ and $V^{(2)} = 2.25$. Note that only two branches are visible because both of them are fourfold degenerate and each energy is shifted by the mean energy of the studied submanifold of eight states. From the figure, we conclude a characteristic 4π periodicity, which implies that these states are not parafermions but weakly coupled Majorana zero modes.

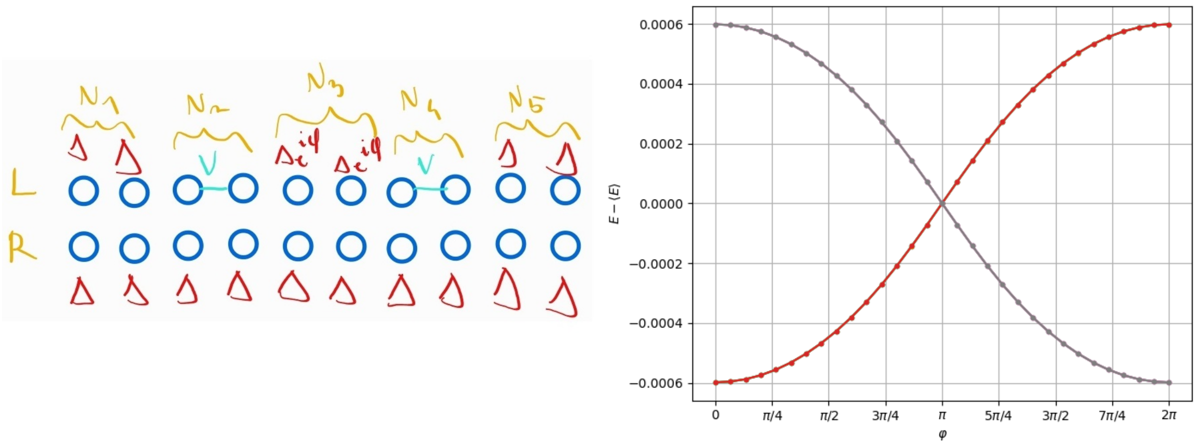

Figure 2: The sketch of a system having four domain wall system with interaction $H_{\text{int}}^{(2)}$ (left), and its Josephson spectrum with parameters $\Delta = 1$, $(N_1, N_2, N_3, N_4, N_5) = (20, 8, 20, 16, 16)$, $V^{(2)} = 2.25$ (right).

3 Publications

In this semester, I was working on my first paper, however, it is still in progress. I also started to examine a different interaction proposed in [7], having the form

$$H_{\mathrm{int}}^{(3)} = \sum_{n,\zeta} V_{n,\zeta}^{(3)} \left[c_{n,\zeta,\uparrow}^{\dagger} c_{n,\zeta,\uparrow} + c_{n,\zeta,\downarrow}^{\dagger} c_{n,\zeta,\downarrow} \right]^2 . \tag{5}$$

However, the calculations using the interaction $H_{\mathrm{int}}^{(3)}$ are still in the early stage. Because of this, there is no figure in this report concerning that interaction, but it can be expected in both the next semester's report and the article.

4 Studies in current semester

I attended two classes in the current semester:

- "Matematikai módszerek a kvantumkémiában I." (subject code: "FIZ/3/034E")
- "Kvantumbitek szilárdtestekben" (subject code: "FIZ/1/041E")

5 Conferences in current semester

I attended the "Lectures on Modern Scientific Programming 2022" conference, hosted by the Wigner Scientific Computing Laboratory.

I am going to attend "American Physical Society's March Meeting", where I will present a contributed talk about my latest results.

6 Teaching activity in current semester

In the current semester, I participated as a lecturer (for 1 class/week, i.e. 2 hours/week) in the practice class "Számítógépes alapismeretek" (with subject codes "szamalapf18la" and "szamalapf19la"), which is an introductory course into Linux basics, LaTeX, and python for first-semester Physics BSc students.

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