Doctoral School of Physics - Eötvös Loránd University (ELTE)

Semester report

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Ph.D. Thesis topic:

Study of solitons and applications with analytical and numerical methods

Personal circumstances:
In this semester, I continue my studies by taking courses entitled “Solitons and Instantons I” and “Non-Equilibrium Statistical Physics” as well as guided research under the supervision of Dr. Bene. Participating these courses, I have the opportunity to observe a wide variety of study methods in my research area. Particularly, in the “Solitons and Instantons I” I learn more about solitons and their application in a wide range of physics. In addition, current researches in this subject were presented by Prof. Palla.

Description of research work carried out in current semester:
The ubiquitous Korteweg de Vries equation is a common approximation for several problems in nonlinear physics. One of these problems is the shallow water wave problem extensively studied during the last fifty years and described in many textbooks and monographs. The KdV equation corresponds to the case when the water depth is constant. There have been numerous attempts to study nonlinear waves in the case of a non-flat bottom. However, the authors did not obtain any simple KdV-type equation. Some researchers obtained an asymptotic solution describing a slowly varying solitary wave above a slowly varying bottom. For small amplitudes, the wave amplitude varies inversely as the depth. Development of packets of surface gravity waves moving over an uneven bottom is studied in kinds of literature.

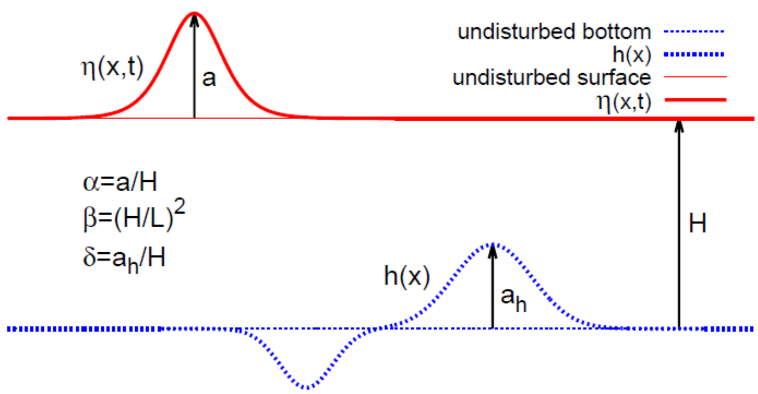

FIG. 1. Schematic view of the geometry of the shallow water wave problem.

In the standard approach to the shallow water wave problem, the fluid is assumed to be inviscid and incompressible and the fluid motion to be irrotational. Therefore a velocity potential ϕ is introduced. It satisfies the Laplace equation with appropriate boundary conditions. The Laplace equation must be valid for the whole volume of the fluid, here as the equations for boundary conditions are valid at the surface of the fluid and at the impenetrable bottom. The system of equations for the velocity potential ϕ, including its derivation, can be found in many textbooks.
In the non-dimensional variables the set of hydrodynamic equations for 2-dimensional flow takes the following form

$$\beta\phi_{xx} + \phi_{zz} = 0$$
$$\eta_t + \alpha\phi_x\eta_x + \frac{1}{\beta}\phi_z = 0 \; for \; z = 1 + \alpha\eta$$
$$\phi_t + \frac{1}{2}\alpha\phi_x^2 + \frac{1}{2}\frac{\alpha}{\beta}\phi_z^2 + \eta = 0 \; for \; z = 1 + \alpha\eta$$
$$\phi_z - \beta\delta(h_x\phi_x) = 0 \;\; for \; z = \delta h(x)$$

The last equation shows bottom variation.

Lagrangian approach
For KdV as it stands, we cannot write a variational principle directly. It is necessary to introduce a velocity potential. The simplest choice is to take $\eta = \varphi_x$ then in the fixed frame takes the form

$$\varphi_{xt} + \varphi_{xx} + \frac{3}{2}\alpha\varphi_x\varphi_{xx} + \frac{1}{6}\beta\varphi_{xxxx} = 0$$

And

$$\mathcal{L}_{fixed} = -\frac{1}{2}\varphi_x\varphi_t - \varphi_x^2 - \frac{\alpha}{4}\varphi_x^3 + \frac{\beta}{12}\varphi_{xx}^2$$

This semester our main concern was working on the Lagrangian approach and the way to study it analytically. To this end, we tried to introduce appropriate trial function and integrate terms by terms to find effective Lagrangian based on collective coordinate method. The trial function was a linear combination of soliton solution of the standard

KdV equation, where widths, heights and positions were left as time dependent variational parameters.

$$\varphi = A_1(t)\,\mathrm{sech}^2[B_1(t) + X_1(t)] + A_2(t)\,\mathrm{sech}^2[B_2(t) + X_2(t)]$$

The governing approximate equations were obtained by varying the action with respect to these parameters. All necessary integrations were carried out numerically via the Gauss-Laguerre method. The resulting ordinary differential equations were also numerically solved. Furthermore, a computer program was written in C++ to study the behavior of the soliton solution asymptotically. Figure 2 shows two soliton interaction.

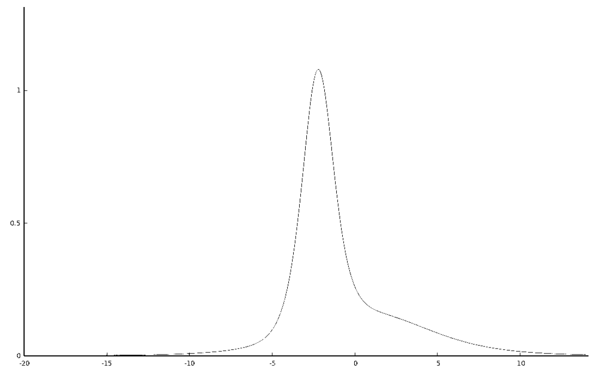

FIG 2. Two soliton interaction at a given time step by C++

Future plans

The next step is to use and validate OpenFOAM as a tool to simulate physical process with uneven bottom. This is another part of the research in which wave generation and wave dynamics will be studied. To this end, there are different scenarios to generate solitary waves and make uneven bottom. The first attempt, in this semester, focused on designing uneven bottoms by using *blockMesh* method in OpenFOAM environment. Figure 3 illustrates two simple bottom shape which are designed by this method.

FIG 3. Simple non-flat bottom

Fortunately, at the end of the semester, our studies was presented in the conference of physics for PhD students called DOFFI. In this presentation, we focused on mathematical aspects of the Lagrangian density and collective coordinate method as well as introducing our basic design in OpenFOAM and its advantages.