Localisation in lattice gauge theory
Semester report: Semester III.

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1 Introduction

The low-lying Dirac modes become localised at the finite-temperature transition in QCD and in other gauge theories, suggesting a general connection between their localisation and deconfinement [1]. According to the sea/islands picture [2], the Dirac operator favours sites where (some of) the phases of the Polyakov loop are close to $\pm\pi$. In $\mathbb{Z}_3$ gauge theory the Polyakov loop phase cannot get closer to $\pm\pi$ than $\pm\frac{2}{3}\pi$ which is the largest minimal distance among $\mathbb{Z}_N$ (and $SU(N)$) gauge models. It was then worth checking whether localisation of the low Dirac modes is present in the deconfined phase of this model, working for simplicity in 2+1 dimensions. Since $\mathbb{Z}_3$ is also the centre of the gauge group of QCD, this may be useful to better understand the possible role played by centre symmetry in the localisation of Dirac modes in that case.

After observing localisation of low modes both in $\mathbb{Z}_2$ and $\mathbb{Z}_3$ gauge theories, there remains little doubt concerning the universality of the connection between deconfinement and localisation properties of Dirac modes. A possible loose end is the case of gauge theories that display a deconfinement transition but whose gauge group has a trivial centre, in which case the finite-temperature transition clearly cannot be associated with the spontaneous breaking of centre symmetry. Localisation of Dirac modes has not been studied yet in such theories, and so it is not clear whether it is present or not. An example for this are $SO(2n+1)$ gauge theories, which I plan to study in the next semester.

2 Research

To study how localisation properties change from the confined to the deconfined phase in the 2+1 dimensional $\mathbb{Z}_3$ lattice gauge theory at finite temperature, one has first to determine the critical temperature, which has not been done yet in the literature. The duality of the model with the 3-state Potts model implies that the transition is second order, and can be exploited to resort to a cluster algorithm [3] to avoid critical slowing down near the transition. Studying the volume scaling of the Binder cumulant, for $N_t = 4$ I found the critical coupling $\beta_c = 1.067(1)$. The results of the finite size scaling analysis are shown in Fig. 1.

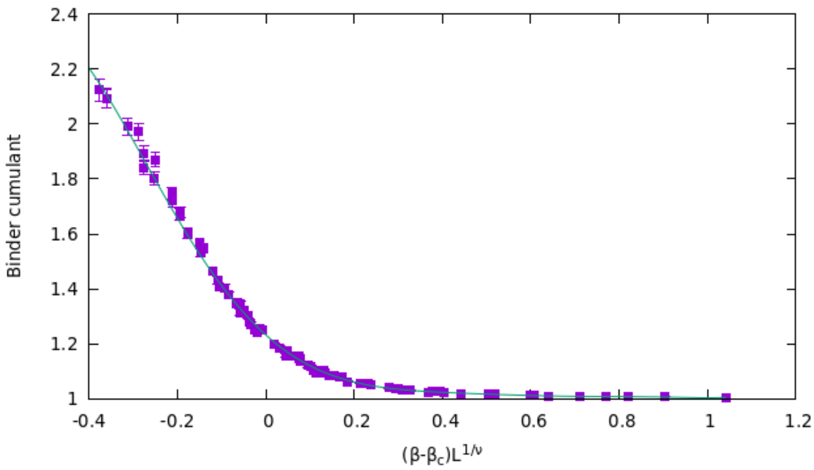

Figure 1: The scaling of the Binder cumulant on $N_t = 4$ lattices. Results from different volumes are plotted against the scaling variable $(\beta - \beta_c)L^{1/\nu}$, where β is the coupling, β_c is the critical coupling, L is the system spatial size and ν is the critical exponent. The result for the critical exponent obtained by fitting the data is $1/\nu = 1.20 \pm 0.35$, which agrees within errors with the expected value $6/5$ [4].

Fig. 2 shows that in the deconfined phase ($\beta > \beta_c$) in the physical (i.e., real) Polyakov loop sector, low modes are localised, as the size of the modes does not increase with the size of the system. This is in agreement with expectations from the sea/islands picture of localisation. The size of the mode is estimated using the participation ratio (PR), i.e., the fraction of volume occupied by the mode, times the system size L^2:

$$\mathrm{PR} \cdot L^2 = \sum_x |\psi(x)|^4.$$

Similarly to what was found in $\mathbb{Z}_2$ gauge theory in 2+1 dimensions, also high modes are localised, both in the confined and the deconfined phase. On the other hand, the size of the bulk modes increases with the spatial length, indicating that they are delocalised.

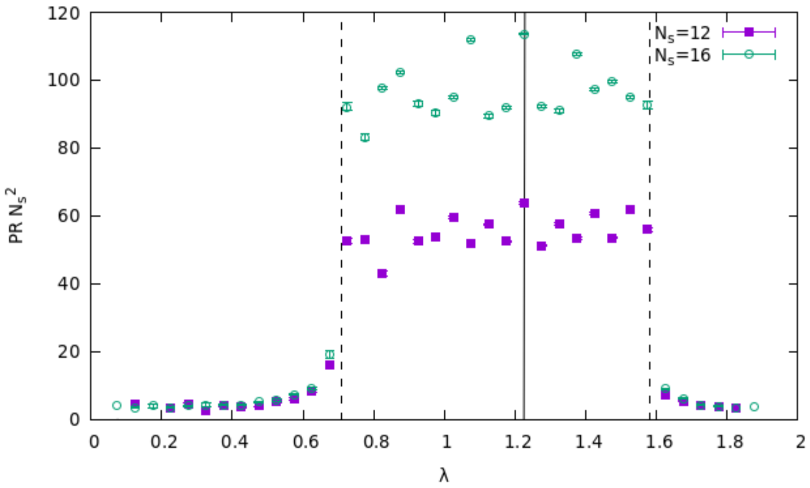

Figure 2: Mode size along the Dirac spectrum for two different volumes (deconfined phase, physical sector).

In Fig. 3 I show the average of the Polyakov loop weighted by the wave function squared of the modes, $\sum_{t,\vec{x}} P(\vec{x})|\psi(\vec{x},t)|^2$. Low and high modes tend to be localised near fluctuations of the Polyakov loop away from order, as shown by the fact that this quantity is well below the average Polyakov loop. For bulk modes instead this quantity is close to the average Polyakov loop, as expected since they are delocalised.

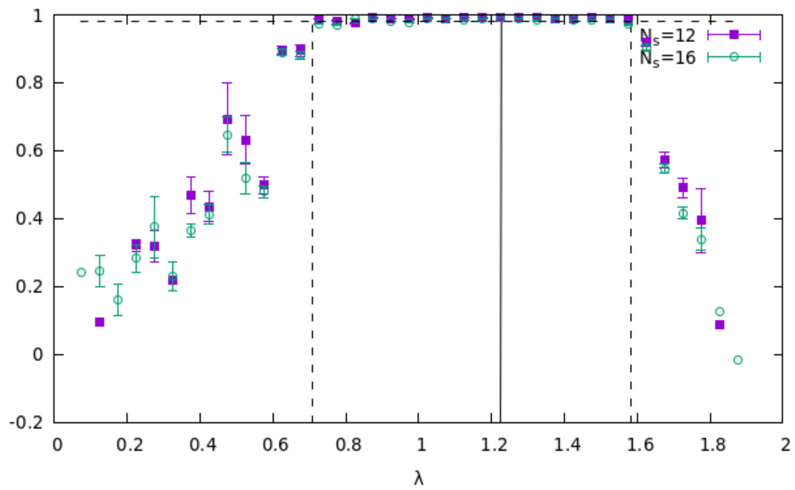

Figure 3: The average Polyakov loop weighted by the wave function along the Dirac spectrum (deconfined phase, physical sector).

3 Publications and conferences

During this semester the manuscript on the localisation properties of eigenmodes of the staggered operator in $\mathbb{Z}_2$ gauge theory (arXiv: 2104.03779), was published in Physical Review D (Phys. Rev. D 104, 054513, 2021).

On July 26-30 I took part in the International Symposium on Lattice Field Theory, where I gave a talk about localisation in $\mathbb{Z}_2$ gauge theory. The proceeding of this talk can be found at arXiv: 2110.15293.

4 Studies

This semester I attended *Solitons and instantons II.*, *Conformal field theory*, *Integrability* and *Renormalization.*

5 Teaching

This semester I took part in teaching the course *Elméleti mechanika* for second year B.Sc. students.

References

[1] Matteo Giordano and Tamás G Kovács. Localization of Dirac fermions in finite-temperature gauge theory. *Universe*, 7(6):194, 2021.

[2] Falk Bruckmann, Tamás G Kovács, and Sebastian Schierenberg. Anderson localization through Polyakov loops: lattice evidence and random matrix model. *Physical Review D*, 84(3):034505, 2011.

[3] Ulli Wolff. Collective Monte Carlo updating for spin systems. *Physical Review Letters*, 62(4):361, 1989.

[4] Rodney J Baxter. *Exactly solved models in statistical mechanics.* Elsevier, 2016.