Localisation in lattice gauge theory
Semester report: Semester I.

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1 Introduction

Confinement and chiral symmetry breaking are the two most striking features of low-energy hadronic physics. Although in principle unrelated, these two phenomena turn out to be closely connected: in fact, at the finite temperature transition of QCD both its confining and chiral properties change dramatically. The nature of the connection between confinement and chiral symmetry breaking is however not fully understood yet. In this respect, an interesting line of investigation is the study of the localisation properties of the low-lying eigenmodes of the Dirac operator. As a matter of fact, these properties change radically at the finite-temperature transition, with the low modes turning from extended to localised as the theory deconfines and chiral symmetry gets effectively restored in the high temperature phase. The same close relationship between deconfinement, chiral symmetry restoration and localisation of the low Dirac modes is found also in other gauge theories. This suggests that the study of localisation in gauge theory can provide clues about the nature and mechanisms of the finite-temperature transition and the relation between confining and chiral properties. The nonperturbative nature of these phenomena makes the lattice approach the most suitable tool for their study.

2 Research done

In my master thesis I studied lattice $\mathbb{Z}_2$ gauge theory in 2+1 dimensions at finite temperature focusing on the spectrum and localisation properties of the staggered Dirac operator. Localisation of low Dirac modes was observed in the deconfined phase even in this simple gauge model. As an extension of my thesis work, I studied the spectrum and localisation properties of the Dirac operator for bigger lattice sizes, focusing particularly on the shape of the eigenmodes.

The shape of the eigenmodes was studied using the inertia tensor obtained by treating $|\Psi(n)|^2$ as the mass of a point particle at point n. Normalisation

of eigenmodes implies that the system has unit mass. For totally delocalised modes one expects the same moment of inertia as a cuboid of size $N_s \times N_s \times N_t$ with N_s the spatial and N_t the temporal size of the system. It is well-known that the moment of inertia of a cuboid of unit mass and uniform density with aspect to the axis passing through its centre and perpendicular to one of its faces, with sides of size a and b, is equal to $\frac{1}{12}(a^2 + b^2)$. This formula has to be modified for a discretised periodic cuboid, with mass concentrated on the sites of a lattice. For lattices of even linear size in all directions I have obtained $\frac{1}{12}((a-1)(a+1)+(b-1)(b+1))$.
As I have shown in my thesis, low and high modes are localised, while bulk modes are delocalised. Figure 1 shows that delocalised modes behave as expected, while low and high modes behave differently. They have much smaller moment of inertia corresponding to smaller size. Based on the relation with the eigenvalues three groups can be separated: Cylinder-shaped (two similar eigenvalues larger than the third), disc-shaped (two similar eigenvalues smaller the the third) and spherical (all eigenvalues are similar) eigenmodes. Figure 1 shows that bulk modes have disc-shape as one would expect, while the low lying modes have cylinder-shape and high modes have spherical shape. This is interesting as it distinguishes between low and high modes which are both localised, and should be investigated further.
Figure 2 shows the orientation of the eigenvectors of the inertia tensor. In the delocalised region the eigenvectors of the inertia tensor are parallel either to the time axis or one of the space axes as they should. For the low modes the main axis of the cylinder is essentially lying on the spatial plane. However, for the high modes the absence of a preferred orientation is consistent with them being spherical.

My future plan is to examine the scaling of the moment of inertia with the volume and finish the investigation of this model. After this I plan to study $SO(2n+1)$ gauge groups. For groups with non-trivial centre, deconfinement is associated with the spontaneous breaking of the centre symmetry. $SO(2n+1)$ groups have trivial centre, so this study will show how relevant is the centre symmetry and its breaking for localisation.

3 Teaching activity

In this semester I took part in teaching the course *Klasszikus fizika laboratórium* (4 hours a week).

4 Studies

I attended *Rácstérelmélet II.* in this semester.

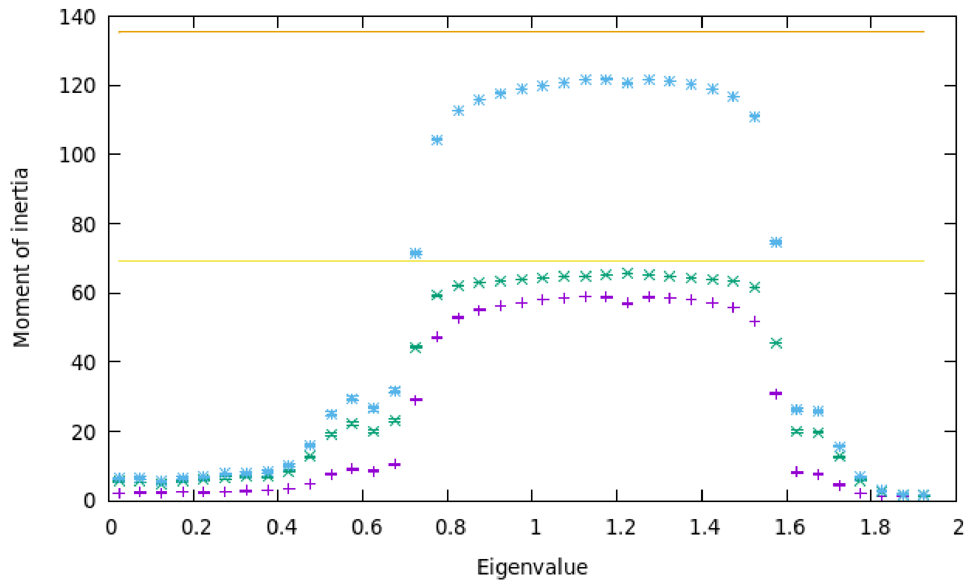

Figure 1: The eigenvalues of the inertia tensor in the spectrum of the Dirac operator. Here $\beta = 0.75$, $N_s = 28$ and $N_t = 4$. The horizontal lines show the theoretical values for the totally delocalised case.

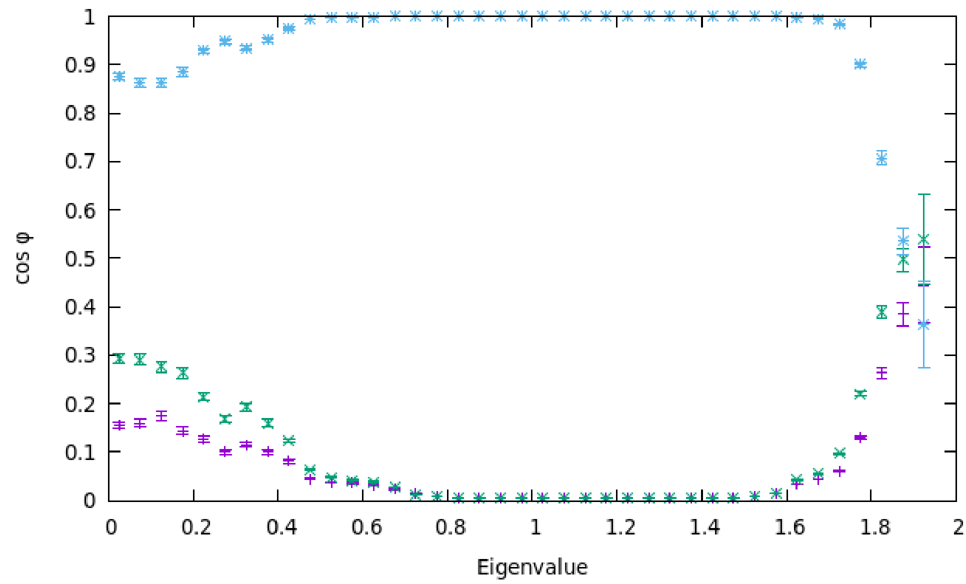

Figure 2: The average orientation of the eigenmodes in the spectrum of the Dirac operator. The colour-code is the same as in Figure 1. Here φ is the angle between the eigenvectors of the inertia tensor and the time direction on the lattice.