Semester report: Semester IV.

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Localisation in lattice gauge theory

1 Introduction

Confinement and chiral symmetry breaking are the two most striking features of low-energy hadronic physics. Although in principle unrelated, these two phenomena turn out to be closely connected: in fact, at the finite temperature transition of QCD both its confining and chiral properties change dramatically. The nature of the connection between confinement and chiral symmetry breaking is, however, not fully understood yet. In this respect, an interesting line of investigation is the study of the localisation properties of the low-lying eigenmodes of the Dirac operator. As a matter of fact, these properties change radically at the finite-temperature transition, with the low modes turning from delocalised to localised as the theory deconfines and chiral symmetry gets effectively restored in the high temperature phase. The same close relationship between deconfinement, approximate chiral symmetry restoration and localisation of the low Dirac modes is found also in other gauge theories. This suggests that the study of localisation in gauge theory can provide clues about the nature and mechanisms of the finite-temperature transition and the relation between confining and chiral properties. The nonperturbative nature of these phenomena makes the lattice approach the most suitable tool for their study.

An intuitive explanation of low-mode localisation is the so-called "sea/islands„ picture [1]. According to this picture, in the deconfined phase of gauge theory the "islands„ of Polyakov loop fluctuations in the "sea„ of ordered Polyakov loops act effectively as "potential wells„ favouring localisation of low Dirac modes. This picture is quite general, and has been verified in several models [2], where correlation between localised modes and Polyakov-loop fluctuations has been observed.

2 Result of previous semesters

To study localisation, the simplest quantity one can look at is the *Participation Ratio* (PR),

$$\mathrm{PR} = \left(\sum_n |\psi(n)|^4\right)^{-1} V^{-1}, \tag{1}$$

averaged over configurations and over modes ψ in a given spectral region. This quantity expresses how extended modes are in the spectral region of interest. When the modes are delocalised, their PR goes to a constant value for large volumes, meaning that the modes occupy the whole lattice. If the modes are localised in a finite region then their PR goes to zero in the infinite volume limit. Equivalently, one can use the mode size, defined as the PR times the lattice size, which diverges as V for delocalised modes and goes to a constant for localised modes. In general, for a mode size scaling like V^{α} one can say that modes have fractal dimension α.

The first model I studied by numerical simulations was the $\mathbb{Z}_2$ gauge model in 2+1 dimensions [3]. Above the deconfinement transition and in the positive Polyakov-loop sector, I found localised modes for the staggered Dirac operator up to a certain point in the spectrum, called “mobility edge„.

The next model I studied was $\mathbb{Z}_3$ gauge theory in 2+1 dimensions. This model is interesting because the ”deepest„ potential wells are the shallowest among $\mathbb{Z}_N$ gauge models in the trivial Polyakov-loop sector, so making localisation harder; and because in the complex Polyakov-loop sectors the islands of fluctuations do not provide potential wells at all, thus making the simplest sea/islands picture unable to predict (and, if it is the case, explain) the localisation of low modes.

As a preliminary task, I had to determine the deconfinement temperature of the model, as it was not available in the literature. This is most efficiently done by exploiting the duality with the 3-state Potts model, for which one can employ a cluster algorithm to avoid critical slowing down near the transition. I determined the critical coupling for lattices of temporal size $N_t = 4$ by studying the Binder cumulant, $\mathscr{B}$, for different spatial sizes. This quantity is volume independent at the critical point, which is then found by looking for the crossing point of curves $\mathscr{B}(\beta)$ for different lattice sizes. The result of this study is shown in Fig. 1. From this one can obtain the critical coupling for the $\mathbb{Z}_3$ gauge theory via the duality relation and get $\beta_c = 1.067(1)$.

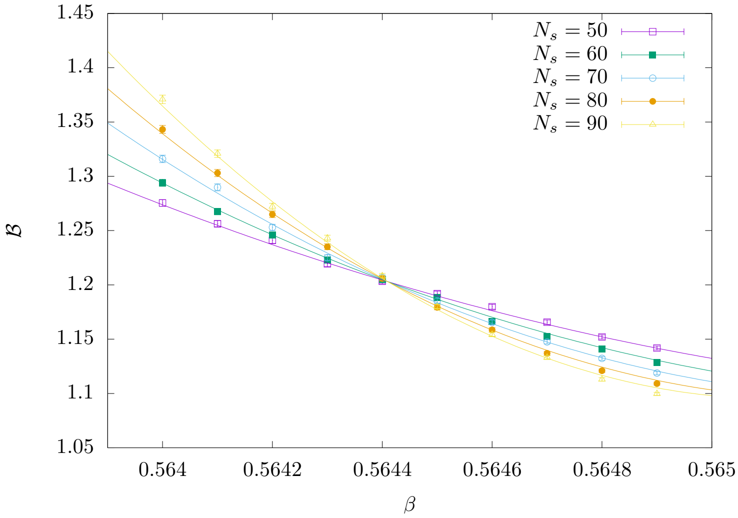

Figure 1: The Binder cumulant computed in the 3-states Potts model.

3 Results in the current semester

After determining the critical temperature, I started full simulation runs in both phases of $\mathbb{Z}_3$ gauge theory for different lattice sizes and temperatures. Below the transition I did not find localisation for low modes (Fig. 2), that have a fractal dimension around 1. For bulk modes the fractal dimension goes closer to 2, meaning that their size scales approximately like the volume. At the high end of the spectrum the fractal dimension drops to zero, meaning that high modes are localised. However, in the deconfined phase in the real Polyakov-loop sector, for low modes the fractal dimension is around zero, showing that these modes are localised; this is the case for high modes as well. The bulk modes are delocalised in the deconfined phase, too. I also confirmed the validity of the sea/islands picture, showing that localised modes correlate with Polyakov-loop fluctuations away from the ordered value.

In the deconfined phase in the complex sector, for low modes the fractal dimension is around zero, showing that these modes are localised (Fig. 4); this is the case for high modes as well, while the bulk modes are delocalised also here.

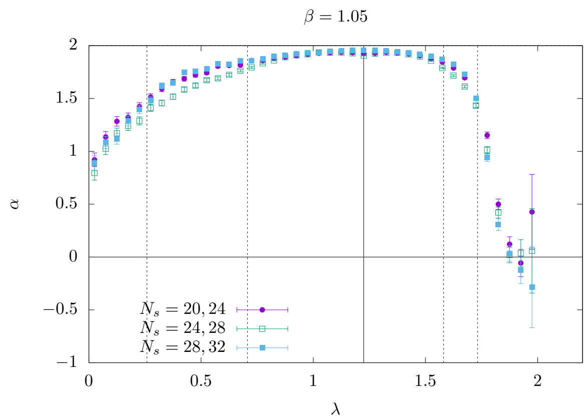

Figure 2: The fractal dimension along the spectrum-confined phase.

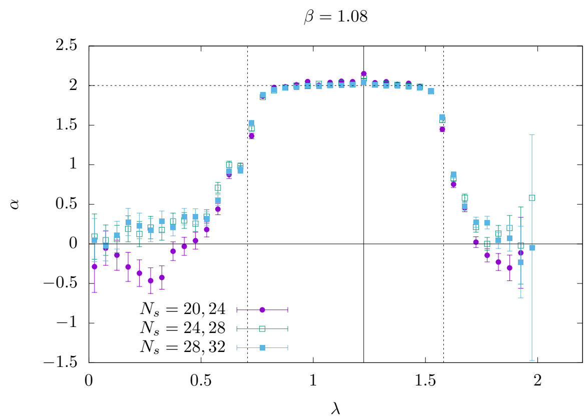

Figure 3: The fractal dimension along the spectrum-deconfined phase, real sector.

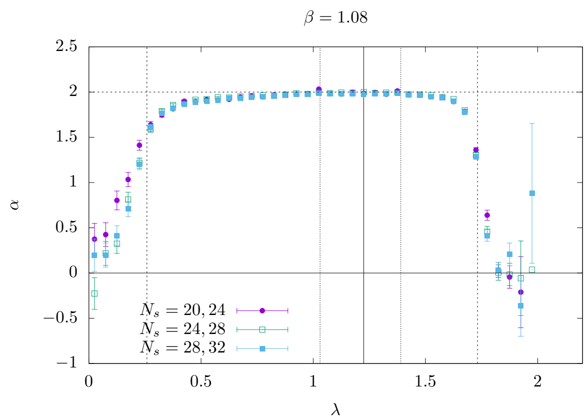

Figure 4: The fractal dimension along the spectrum-deconfined phase, complex sector.

4 Courses

In the semester I attended *Advanced field theory*.

5 Publications

Localisation properties of eigenmodes of the staggered operator in $\mathbb{Z}_2$ gauge theory (arXiv: 2104.03779), was published in Physical Review D (Phys. Rev. D 104, 054513, 2021).

Deconfinement transition and localization of Dirac modes in finite-temperature $\mathbb{Z}_3$ gauge theory on the lattice, writing is in progress.

6 Conferences

In 2021 on July 26-30 I took part in the International Symposium on Lattice Field Theory, where I gave a talk about localisation in $\mathbb{Z}_2$ gauge theory. The proceeding of this talk can be found at arXiv: 2110.15293.

7 Teaching activity

In the current semester I taught *Elektromágnesség* to BSc students (1 hour per week).

References

[1] Falk Bruckmann, Tamás G Kovács, and Sebastian Schierenberg. Anderson localization through Polyakov loops: lattice evidence and random matrix model. *Physical Review D*, 84(3):034505, 2011.

[2] Matteo Giordano and Tamás G Kovács. Localization of Dirac fermions in finite-temperature gauge theory. *Universe*, 7(6):194, 2021.

[3] György Baranka and Matteo Giordano. Localization of dirac modes in finite-temperature $\mathbb{Z}_2$ gauge theory on the lattice. *Physical Review D*, 104(5):054513, 2021.