1. Semester report №4

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Ph.D. Thesis title: Searching for signs of new physics beyond the standard model

Introduction: I am a second-year international student at ELTE-Doctoral School of Physics from the 2020/2021 Autumn semester. In first semester, because of some difference between the Mongolian and Hungarian educational systems, I had to catch up on more advanced topics first and did not start my research work. However, 2nd semester I started my research work which is based on cosmological aspect of Super-weak U(1) extension of Standard Model (SWSM) [5]. In particular, I am interested in late-time accelerated expansion of the universe and its possible connection to the inflation.

The Nobel Prize winning confirmation in 1998 of the accelerated expansion of our universe put into sharp focus the need of a consistent theoretical model to explain the origin of this acceleration [3-4]. The term "dark energy" coined responsible for this accelerated expansion of the universe and over the past decades there has been a huge theoretical and observational effort into improving our understanding of the universe [1]. Unfortunately, we still do not have consistent cosmological model responsible for the late-time accelerated expansion of the universe and, it is shown that the models with canonical scalar field or non-minimal coupling with scalar field and gravitational field can be produce the accelerated expansion of the universe [1-2]. In the cases of the multiple scalar fields, a very small number of research is done [1], since with multiple scalar fields, the phase space of the dynamics of the universe extended largely and becomes complicated. However, some models of the extension of the Standard model, includes second scalar field. Especially, SWSM has two scalar fields, one is a Higgs field and second is responsible for the super-weak symmetry breaking. So, it is possible that one of them or both can be producing the late-time accelerated expansion of the universe.

In order to study the dynamical evolution of the universe, we used the dynamical system analysis (DSA) approach. The cosmological equations representing the dynamics of a homogeneous and isotropic universe are systems of ordinary differential equations, and one of the most elegant ways these can be investigated is by casting them into the form of dynamical systems [1]. This allows the use of powerful analytical and numerical methods to gain a quantitative understanding of the cosmological dynamics derived by the models under study.

I am a member of ELTE – Particle Phenomenology Group, which has 8 members and supervised by professor Zoltán Trócsányi. Currently, there are many ongoing research topics, such as Cosmological constraints on the U(1) extension of Standard Model, Neutrino mass model, and Vacuum stability. My current research interest is a connection between cosmology, inflation and particle physics.

Summary of research work carried out in the previous three semesters: My current research work mostly concerns with the cosmological aspect of SWSM. The SWSM has many phenomenological applications such as: inflation, neutrino mass and late-time accelerated expansion of the universe. As the SWSM has two scalar fields, it might be responsible for inflation and dark energy. In order to study late-time accelerated expansion of the universe in the SWSM, we are using the dynamical system analysis (DSA) approach. The dynamical system analysis is based on developing autonomous system equations from given Lagrangian density and studying critical points of that system. In other words, DSA treats the Friedmann equations as autonomous system equations and analyzes its critical points. To analyze the stability of the critical points of the system, the DSA needs different methods. The standard method is to use the linear stability theory, which is based on slight perturbations of the critical points. However, the linear stability theory fails to analyze the stability of the critical points when the eigenvalue of the Jacobian of the system has vanishing real part. In this case, the center manifold theory or the Lyapunov function method is used, but we are mostly stick with the center manifold theory, since it is more analytical way to analyze the stability of the given system.

In the first three semester, I was mostly learning the DSA and the stability analysis. In particular, I was studying the standard cosmological model and, cosmological models with one scalar field. From the DSA of the standard cosmology with cold dark matter and cosmological constant, we can see that our universe is starts from radiation dominated universe, passes through matter dominated phase then eventually approaches the dark energy dominated phase, see Figure 1.

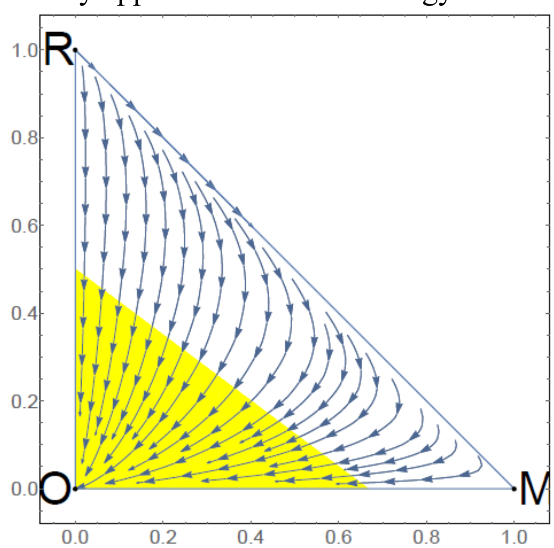

Figure 1: The phase space portraits of the standard cosmology. The yellow shaded area denotes the region of the phase space where the universe is accelerating.

Consequently, a viable cosmological model (theory) might produce a universe which undergoes dynamics shown in the Figure 1. It is possible to produce phase space in the Figure 1 by simply adding a new scalar field to the Einstein-Hilbert action, these models collectively named *Quintessence models.*

In the standard cosmology, the cosmological constant has a role for the dark energy, and for the fixed amount it can give us the late-time accelerated expansion of the universe. But why? This fine-tuning problem is called "cosmological problem" [1]. However, by adding a scalar field with minimal coupling to gravitational field, we do not need the cosmological constant. The scalar field will act like a cosmological constant with its own dynamics. More precisely, when the universe is dominated by the potential energy of the scalar field, it will undergo the accelerated expansion. So late-time dynamics of the universe is defined from the form of the potential.

Generically, we can choose two types of the potential for the scalar field: (i) exponential potential (ii) power-law potential. The latter case is closer the standard model of particle physics, but its stability cannot be determined from the linear stability theory. In the exponential case, linear stability theory can give us full picture of the phase space. The table 1 shows the critical points of the cosmological dynamic system with an exponential potential, while table 2 shows those for a power-law potential.

Point	x	y	Existence
O	0	0	any
$A_\pm$	± 1	0	any
B	$\frac{\sqrt{\frac{3}{2}}(\omega+1)}{\lambda}$	$\frac{\sqrt{\frac{3}{2}}\sqrt{(1-\omega^2)}}{\lambda}$	$\lambda^2 \geq 3(1+\omega)$
C	$\frac{\lambda}{\sqrt{6}}$	$\sqrt{1-\frac{\lambda^2}{6}}$	$\lambda^2 < 6$

Table 1: The critical points of the system when the potential has exponential form.

Point	x	y	z	ω_{eff}	Existence
O_z	0	0	always	ω	always
$A_\pm$	± 1	0	0	1	always
B_x	always	0	1	$\omega + x^2(1-\omega)$	always
C	0	± 1	0	-1	always

Table 2: The critical points of the system with power-law potential.

In the both cases, the fixed (critical) point C is responsible for the late-time accelerated expansion of the universe. Their stability is given by table 3 and table 4.

Point	Stability
O	Saddle
A_+	Unstable if $\lambda^2 < 6$, saddle if $\lambda^2 > 6$.
A_-	Unstable if $\lambda^2 > -6$, saddle if $\lambda^2 < -6$.
B	Stable node if $3(\omega+1) < \lambda^2 < \frac{24(\omega+1)^2}{9\omega+7}$, stable spiral if $\lambda^2 \geq \frac{24(\omega+1)^2}{9\omega+7}$
C	Stable if $\lambda^2 < 3(1+\omega)$, saddle if $3(1+\omega) > \lambda^2 > 6$.

Table 3: The critical points of the system when the potential has exponential form.

Point	Eigenvalues
O_z	$\{ 0 , -\frac{3}{2}(\omega \pm 1)(z-1) \}$
$A_\pm$	$\{0, 3, 3-3\omega\}$
B_x	$\{ 0, -\frac{3}{2}x, \sqrt{6}(\Gamma - 1)x \}$
C	$\{$ -3, 0 , $-3(1+\omega) \}$

Table 2: Eigenvalues of the critical points of the system with power-law potential. Since it has vanishing eigenvalues, the linear stability theory failed to analyze the stability of the system with a power-law potential.

When the potential has power-law form, the linear stability theory fails to analyze the stability of the critical points of the system since Jacobian of the system has vanishing real part. To analyze the stability of the critical points of the system with power-law potential, we used center manifold theory. The method of center manifold theory reduces the dimensionality of the dynamical system, and the stability of this reduced system can then be investigated. The stability properties of the reduced system determine the stability of the critical points of the full system [1]. In the result, the critical point C is stable or unstable depending on the sign of the power being negative or positive in the power-law potential. The power-law potential with negative power can predict late time accelerated expansion of the universe, but with positive power it cannot do so.

In conclusion, the cosmological models with canonical scalar fields, namely, quintessence models with exponential potential or inverse power-law potential can predict the dark energy dominated universe [1]. Unfortunately, these models are not consistent with the standard potential of particle physics, in particular with the potential of the SWSM. However, in the SWSM, with 2 scalar fields and with their mixings, we have more degrees of freedom than in the case of the single field quintessence models. Also, another possibility is to add non-minimal kinetic coupling.

Research work carried out in the current semester: In the previous semesters, I was mostly working with quintessence models with the single scalar field case. In other words, my research mostly covered the cosmological models with single canonical scalar field and reproduced the already standard results in the literature [1-2]. In the case of multiple scalar fields, the most common models are named “assisted quintessence models” [1]. The assisted quintessence models can predict inflation, but do not typically discuss late-time acceleration. However, it is possible to extend them to late-time dynamics of the universe. The simplest case of assisted quintessence model is where 2 canonical scalar fields with exponential potential are present. The dynamical system with 2 canonical scalar fields with exponential potential has a 4-dimensional phase space and 2 independent coupling parameters. One important aspect of the assisted quintessence model is that while none of the single scalar fields can predict the late-time accelerated expansion of the universe, their average determining their combined potential can produce the dark energy dominated phase. The only question is whether the critical points representing the dark energy dominated phase is stable.

The assisted quintessence model with 2 exponential potentials has 13 different fixed (critical) points and only 4 of them can represent the dark energy dominated phase of the universe. The most trivial solutions (fixed points) are where one of the scalar fields is completely vanishes, these fixed points are shown in the table 5 and their stability is shown in the table 6. One important difference from the single field case is that now there are critical points where one of the scalar fields vanish, hence it may resemble the single-field case when such a point is a stable attractor. In the two-field case however, such a critical point becomes a saddle point. Meaning that, assisted quintessence models with 2 non-interacting exponential potentials cannot produce the late-time accelerated expansion of the universe [1].

P	x_1	x_2	y_1	y_2	Exist.	ω_{eff}	Accel.
O_M	0	0	0	0	always	0	No.
C_{ϕ_1}	$\frac{\lambda_1}{6}$	0	$\mp\sqrt{1-\frac{\lambda_1^2}{6}}$	0	$\lambda_1 \leq \frac{1}{\sqrt{6}}$	$\frac{1}{3}(-3+\lambda_1^2)$	$\lambda_1 \leq \sqrt{2}$
C_{ϕ_2}	0	$\frac{\lambda_2}{6}$	0	$\mp\sqrt{1-\frac{\lambda_2^2}{6}}$	$\lambda_2 \leq \frac{1}{\sqrt{6}}$	$\frac{1}{3}(-3+\lambda_2^2)$	$\lambda_2 \leq \sqrt{2}$

Table 5: The critical points of the assisted quintessence model with exponential potential. Here, only the trivial solutions are shown, where one of the scalar fields is completely vanished.

P	Eigenvalues	Stability
C_{ϕ_1}	$\{\frac{\lambda_1}{2}, \frac{1}{2}(6-\lambda_1^2), -\frac{1}{2}(6-\lambda_1^2), -3+\lambda_1^2\}$	Saddle.
C_{ϕ_2}	$\{\frac{1}{2}(-6+\lambda_2^2), \frac{1}{2}(-6+\lambda_1^2), -\frac{\sqrt{-3\lambda_2^2+\lambda_2^4}}{\sqrt{2}}, \frac{\sqrt{-3\lambda_2^2+\lambda_2^4}}{\sqrt{2}}\}$	Saddle

Table 6: The stability of the critical points. Neither of them is stable attractor.

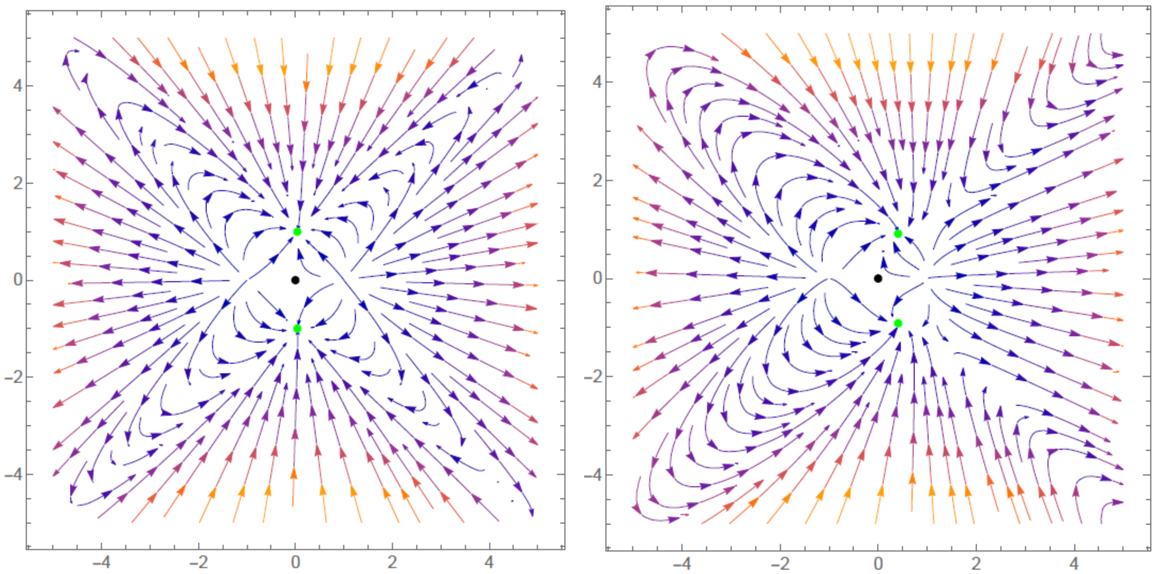

Figure 4: The phase space portraits of the critical points (green dots) with different values of the coupling. The critical points on the left picture looks like stable attractor but on the right it is not where the coupling is much bigger.

There are other two fixed points, where both scalar fields are present. It is difficult to analyze their stability by analytically because the expressions corresponding to the eigenvalues are too long and Mathematica software could not find suitable analytical solutions.

P	x_1	x_2	y_1	y_2	ω_{eff}	Accel.
O_M	0	0	0	0	0	No.
$C_{\mp}$	$\frac{\lambda_1\lambda_2^2}{\sqrt{6}(\lambda_1+\lambda_2)}$	$\frac{\lambda_1^2\lambda_2}{\sqrt{6}(\lambda_1+\lambda_2)}$	$\mp\frac{\sqrt{\frac{6\lambda_2^4-\lambda_1^2\lambda_2^2(-6+\lambda_2^2)}{\lambda_1+\lambda_2}}}{\sqrt{6}\sqrt{\lambda_1^2+\lambda_2^2}}$	$\mp\frac{\sqrt{6\lambda_1^2-\lambda_1^4+\frac{\lambda_1^6}{\lambda_1^2+\lambda_2^2}}}{\sqrt{6}\sqrt{\lambda_1^2+\lambda_2^2}}$	$-1+\frac{\lambda_1^2\lambda_2^2}{3(\lambda_1^2+\lambda_2^2)}$	$0<\lambda_2\le\sqrt{2}$ with $\lambda_1>0$ or $\lambda_2>\sqrt{2}$ with $0<\lambda_1<\sqrt{2}\sqrt{\frac{\lambda^2}{-2+\lambda^2}}$
$\tilde{C}_{\mp}$	$\frac{\lambda_1\lambda_2^2}{\sqrt{6}(\lambda_1+\lambda_2)}$	$\frac{\lambda_1^2\lambda_2}{\sqrt{6}(\lambda_1+\lambda_2)}$	$\mp\frac{\sqrt{\frac{6\lambda_2^4-\lambda_1^2\lambda_2^2(-6+\lambda_2^2)}{\lambda_1+\lambda_2}}}{\sqrt{6}\sqrt{\lambda_1^2+\lambda_2^2}}$	$\pm\frac{\sqrt{6\lambda_1^2-\lambda_1^4+\frac{\lambda_1^6}{\lambda_1^2+\lambda_2^2}}}{\sqrt{6}\sqrt{\lambda_1^2+\lambda_2^2}}$	$-1+\frac{\lambda_1^2\lambda_2^2}{3(\lambda_1^2+\lambda_2^2)}$	$0<\lambda_2\le\sqrt{2}$ with $\lambda_1>0$ or $\lambda_2>\sqrt{2}$ with $0<\lambda_1<\sqrt{2}\sqrt{\frac{\lambda^2}{-2+\lambda^2}}$

Table 7: The fixed (critical) points representing both scalar fields are present. They can be achieving the late-time accelerated expansion of the universe, depending on the certain conditions.

So, to study the stability of this points, we did parameter scan for the given range. The range is $\{10^{-5}, 5\}$ with 10 000 iterations and assumed the couplings (λ's) are real and positive. Within this parameter scan, the critical points were saddle points and it is unlikely to be stable points with more range. The phase space portrait is shown in the Figure 4.

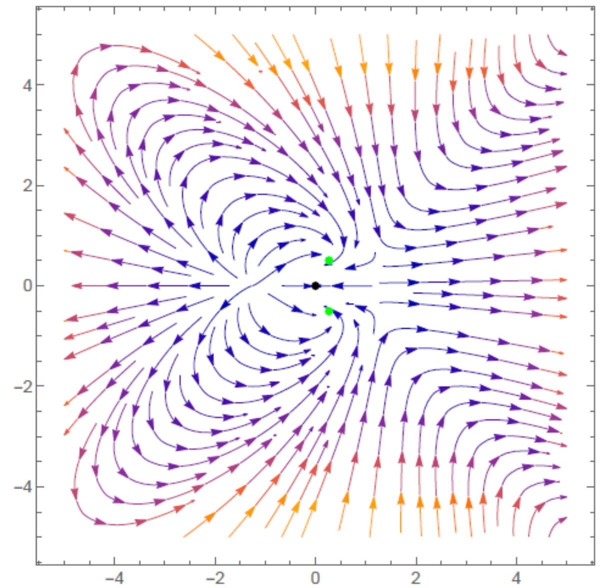

Figure 4: The phase space portrait of the critical points with certain of values of the coupling within the scan range. From phase space it is obvious that the fixed points (green dots) are saddle points. The black dot represents the matter dominated phase of the universe.

Next case was with power-law potential. With two power-law potentials, the dynamical system has 30 critical points, and 15 of them can satisfy the condition for late-time accelerated expansion of the universe. However, it is difficult to study their stability, since now system has a singular Jacobian, meaning that dynamics of the equations cannot be separated into eigenvector base system. Moreover, the system has at least 2 vanishing eigenvectors, so it is impossible to use linear stability theory. Instead of linear stability theory, we used center manifold approach. But there are still difficulties, because the center manifold describing the dynamics of the critical points, is now at least 3-dimensional surface where it is hard to tell the stability of the fixed points, but not hopeless. I have not finished this analysis, but I expect to finish before July.

By using the center manifold approach, we also studied the gravitational theory with the Gauss-Bonnet term and I expect to publish the results before August. My contribution to this research work is to analyze the stability of the critical points of the system, especially the critical points representing the late-time accelerated expansion. (see 2^{nd} publication plan).

Publications:

1. Bilguun Bayarsaikhan, Gansukh Tumurtushaa, Seoktae Koh, Enkhbat Tsedenbaljir. "*Constraints on dark energy models from Horndeski theory*". JCAP 11 (2020) 057;
2. [In-preparation 70%] Sunly Khimphun, Pearun Ritchy, Gansukh Tumurtushaa, Bilguun Bayarsaikhan. *Dynamical analysis with Gauss-Bonnet and Non-minimal coupling in 4D.*

Studies in current semester:

1. **FIZ/02/016E** Finite temperature quantum field theory and astrophysical applications
2. **FIZ/02/002E** Standard model
3. **FIZ/02/001E** Advanced field theory

References

[1] Sebastian Bahamonde, Christian G. Boehmer, Sante Carloni, Edmund J. Copeland, Wei Fang, Nicola Tamanini. *Dynamical systems applied to cosmology: dark energy and modified gravity.* Physics Reports Vol. 775-777 (2018) 1-122.

[2] Jiro Matsumoto, Sergey V. Sushkov. *Cosmology with nonminimal kinetic coupling and a Higgs-like potential.* JCAP 1511, no. 11, 047 (2015).

[3] A.G. Riess et al., *Observational evidence from supernovae for an accelerating universe and a cosmological constant.* Astron. J., Vol., **116**, 1009-1038 (1998).

[4] S. Perlmutter et al., *Measurement of \Omega and \Lambda from 42 high-redshift supernovae.* Astrophys.J., **517** 565-586, (1999).

[5] Zoltan Trocsanyi. *Super-weak force and neutrino masses.* arXiv: 1812.11189v2.