Semester report

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Ph.D. Thesis title: Investigation of topological superconductors in the presence of interactions

1 Research work carried out in current semester

During this semester, I worked on the previously introduced system, trying to figure out the reason behind the 4π-periodic Josephson spectrum.

1.1 Current model

As I introduced it in the previous report, my ladder-like kinetic Hamiltonian term can be written as

$$\begin{aligned} H_{\text{kin}} = &\sum_{n,\sigma} \begin{bmatrix} c^\dagger_{n,L,\sigma} & c^\dagger_{n,R,\sigma} \end{bmatrix} \begin{bmatrix} -\mu & t \\ t & -\mu \end{bmatrix} \begin{bmatrix} c_{n,L,\sigma} \\ c_{n,R,\sigma} \end{bmatrix} \\ &- \frac{t}{2} \sum_{n,\sigma} \left(\begin{bmatrix} c^\dagger_{n+1,L,\sigma} & c^\dagger_{n+1,R,\sigma} \end{bmatrix} \begin{bmatrix} i\sigma & 1 \\ 1 & -i\sigma \end{bmatrix} \begin{bmatrix} c_{n,L,\sigma} \\ c_{n,R,\sigma} \end{bmatrix} + \text{h.c.} \right), \end{aligned} \tag{1}$$

where t is a real parameter, μ us the chemical potential, $n \in \mathrm{N}$ is the site degree of freedom, $\zeta \in \{\mathrm{L}, \mathrm{R}\}$ is the newly introduced "leg" degree of freedom (that is, our model can be imagined as a ladder), and σ is the spin (with $\uparrow = 1$, $\downarrow = -1$ convention).

The s-wave superconductivity is taken into account via Cooper-pair creation and annihilation, that is

$$H_{\text{sc}} = \sum_{n,\zeta} \Delta_{n,\zeta} \left[c^\dagger_{n,\zeta,\uparrow} c^\dagger_{n,\zeta,\downarrow} + \text{h.c.} \right]. \tag{2}$$

In the previous semester, I examined two different interactions and planned to work with a third one as well, but at the start of this semester, I reached promising result with the interaction $H^{(2)}_{\text{int}}$ [1]. Because of this, my only focus was on the time-reversal symmetry and parity conserving interaction

$$H_{\text{int}} = \sum_{n,\zeta} V_{n,\zeta} \left[c^\dagger_{n,\zeta,\uparrow} c_{n,\zeta,\downarrow} c^\dagger_{n+1,\zeta,\uparrow} c_{n+1,\zeta,\downarrow} + \text{h.c.} \right], \tag{3}$$

where $V_{n,\zeta}$ is the interaction strength, and I dropped the index 2 for simplicity.

I also used an x-directional magnetic field during my calculations, which can be written as

$$H_{\text{B}} = \sum_{n,\zeta} B_{n,\zeta} \left[c^\dagger_{n,\zeta,\uparrow} c_{n,\zeta,\downarrow} + \text{h.c.} \right]. \tag{4}$$

1.2 Different phase diagrams

I already presented a phase diagram in the previous semester, but I looked into several different ones to deepen my understanding of this system.

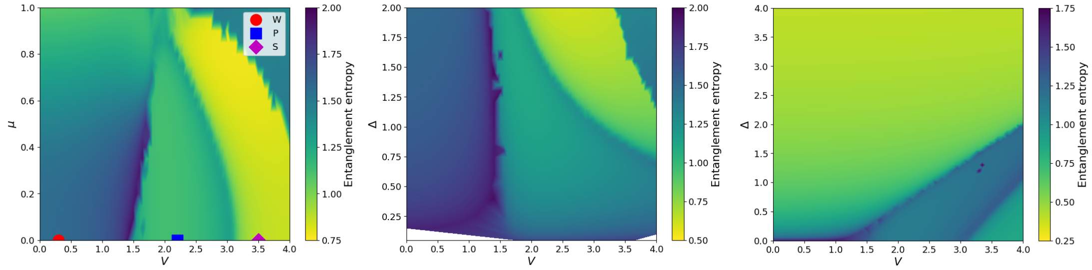

Figure 1: Different phase diagrams of the system. All of them have lengths $N_1 = N_3 = 20$, $N_2 = 100$, with $t = 1$, superconducting right leg and edges on the left leg (with lengths N_1 and N_3). Left: interaction on the midde of the left leg, and $\Delta = 1$ on the superconducting regions. Middle: The same system as on the left, but with $\mu = 0$. Right: The superconductivity on the right leg and at the edges of the left leg are fixed at $\Delta = 1$, and its varying on the middle region, where the interaction is present.

In Figure 1, three different phase diagrams can be seen. All three of them are quite interesting, but in this report I am going to focus on the left one, as that is the one I examined for the longest amount of time. However, the other two are also interesting, and I am planning to focus on them in the future.

In this phase diagram, there are three interesting regions: the weakly interacting (W), the strongly interacting (S) and the fourfold degenerate phase, with possibly parafermionic excitations (P). In Figure 2, the excitation spectrum of the three interesting regions are shown as a function of the system length.

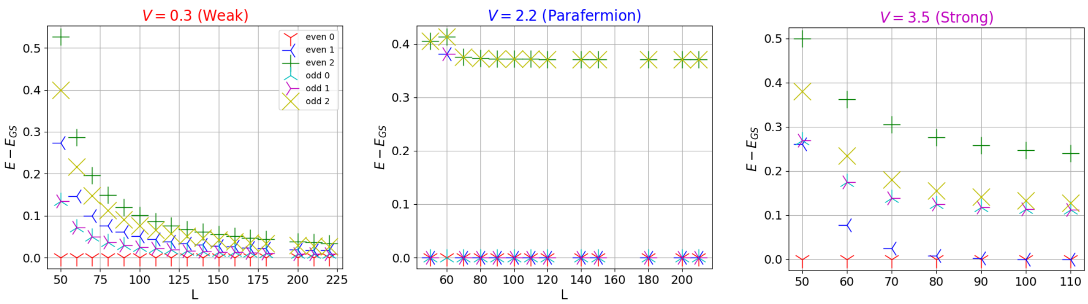

Figure 2: The excitation spectrum of the three interesting regions. In every case, $\Delta = 1$, $t = 1$, $N_1 = N_3 = 20$ and $N_2 = N - N_1 - N_3$.

1.3 Josephson spectrum

In the previous semester, I achieved a 4π-periodic Josephson spectrum with the aforementioned interaction, which can be seen in Figure 3. This result was quite surprising, because I expected it to be 8π-periodic. Because of this, I tried to look deeper into it.

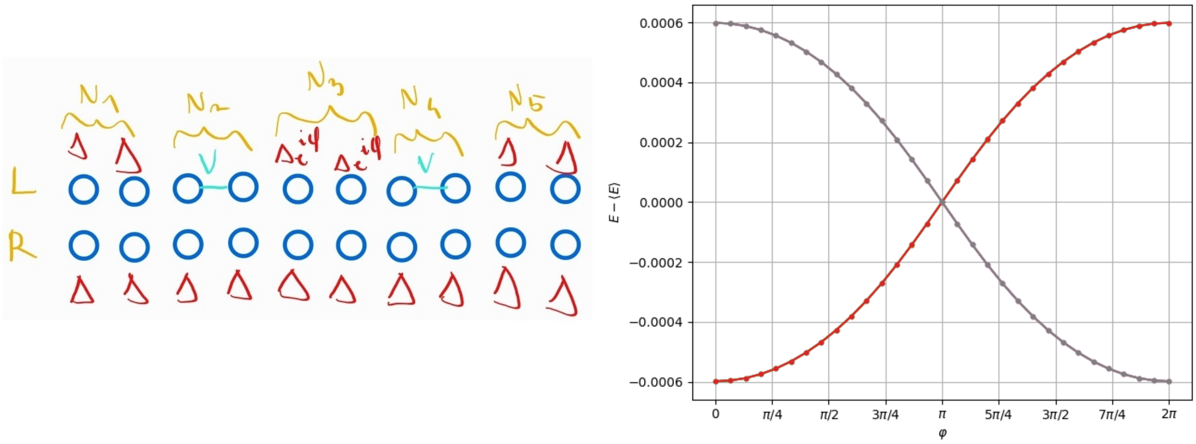

Figure 3: The sketch of a system having four domain wall system with interaction H_{int} (left), and its Josephson spectrum with parameters $\Delta = 1$, $(N_1, N_2, N_3, N_4, N_5) = (20, 8, 20, 16, 16)$, $V = 2.25$ (right).

In the current semester, the setup changed a little bit: On the left leg, the fourth section's interaction was replaced by magnetic field in the x direction, to make the numerical calculations easier. In the previous section there are 4 parafermions, making the ground state $2^4 = 16$-fold degenerate. Replacing two of those with Majorana fermions lowers the degeneracy to $2 \times 2^2 = 8$-fold.

In Figure 3, the points are perfectly lying on an analytical curve, however, I noticed that with higher calculational precision, some of the points start to deviate a little bit from that line. Because of this, I tried to diagonalize the degenerate subspace after the calculations. If the resulting vectors from the DMRG calculation are labeled as $\{\chi_i\}_{i=1}^{k}$ and the Hamiltonian of the system is denoted by H, the effective Hamiltonian matrix can be written as $h_{i,j} = \langle \chi_i | H | \chi_j \rangle$ and the effective overlap matrix as $s_{i,j} = \langle \chi_i | \chi_j \rangle$. Then, the generalized eigenvalue problem for the k-th eigenvector v_k and eigenvalue e_k reads as

$$h v_k = e_k s v_k. \tag{5}$$

Note that ideally, s would be the identity matrix, but due to the deviation from that, it results in a small contribution that breaks the spectrum further.

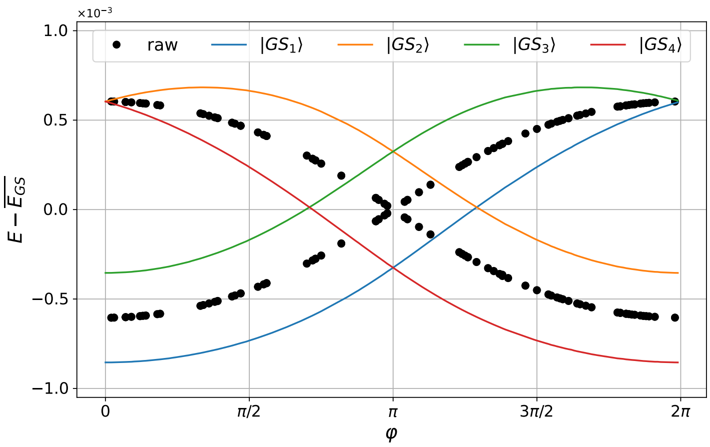

Figure 4: The new josephson spectrum with interaction H_{int}, with parameters $\Delta = 1$, $t = 1$, $(N_1, N_2, N_3, N_4, N_5) = (20, 8, 20, 16, 16)$, $V = 2.2$ (right). The "raw" points are the output of the DMRG, while the solid lines are the result of the orthogonalization.

In Figure 4, the Josephson spectrum is plotted before the orthogonalization (i.e. the "raw" data) and after it. Note that the "raw" points look like before, but the solid lines are clearly 8π-periodic, indicating that they are parafermions.

2 Publications

For my first paper, all of the calculations are done and the writing is in progress, so I expect it to be submitted in a few weeks.

3 Studies in current semester

I attended two classes in the current semester:

- "Haladó statisztika és modellezés" (subject code: "FIZ/3/088")
- "Kvantuminformáció-elmélet" (subject code: "FIZ/3/060E")

4 Conferences in current semester

I attended the "American Physical Society's March Meeting 2023" (2023.03.05-10., Las Vegas), where I presented a contributed talk about my latest results.

I attended the "Quantum National Laboratory Workshop" (2023.06.09., Budapest), where I presented my latest result with a poster.

I attended "AndQC - TOPSQUAD workshop: Bound states in superconducting nanodevices" (2023.06.12-14., Budapest), where I presented my latest result with a poster.

5 Teaching activity in current semester

In the current semester, I participated as a lecturer (for 1 class/week, i.e. 2 hours/week) in the practice class "A fizika numerikus módszerei 1" (with subject code "fiznum1f19la"), which is a course about useful python tools in physics for second-semester Physics BSc students.

6 Acknowledgements

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I acknowledge KIFÜ for awarding me access to computational resources based in Hungary.

References

[1] Christoph P. Orth, Rakesh P. Tiwari, Tobias Meng, and Thomas L. Schmidt. Non-abelian parafermions in time-reversal-invariant interacting helical systems. *Phys. Rev. B*, 91:081406, Feb 2015.