ELTE

TTK

Semester report - II.: Connecting inflation and dark energy with particle physics

Particle and Nuclear Physics program

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1 Introduction

In this semester I continued my work with scalar field cosmology, motivated by Lagrangians based on particle physics. The overall aim is to compute for a given model the spectral parameters n_s (the spectral index) and r (the scalar-tensor ratio) and compare them to observations in the CMB power spectrum [1]. This requires two steps: computing the temporal evolution of the homogeneous background, and relating it to the power spectrum via the theory of cosmic perturbations.

The short term goal was to make progress in these two areas. For one, I derived the equations governing the background, and could solve them for arbitrary parameters and initial conditions numerically. The solutions need to be validated, and the regimes where inflation occurs need to be found. On the other hand, the power spectrum generated by conformally coupled scalars is rarely found in the literature. I aimed to derive them, and compare them to the few sources I found.

2 Guided research

2.1 Temporal evolution

I derived the equations of motion describing the temporal evolution of a flat FLRW universe with its sole energy contributions being scalar fields:

$$\vec{\varphi}'' = -(3-\varepsilon_1)\left[\vec{\varphi}' + \frac{\partial \ln \kappa^2 V(\{\varphi_i\})}{\partial \vec{\varphi}}\right], \tag{2.1}$$

where the scalars are arranged in a vector $\vec{\varphi}$, κ is the usual gravitational constant, a $'$ denotes differentiation with regards to the e-folds η, and

$$\varepsilon_1 = \frac{1}{2}\vec{\varphi}'^2, \tag{2.2}$$

which is the first Hubble slow-roll parameter. It allows us to single out regions where the expansion of the universe is accelerating, i. e. inflation occurs.

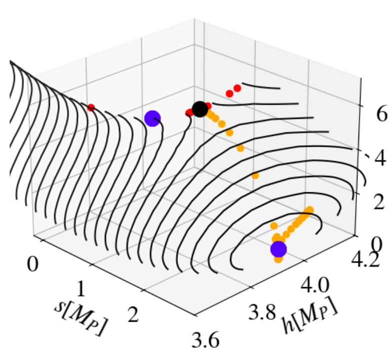

Figure 1: Temporal evolution of two scalar fields along their potential, with their vacuum expectation values in blue, and the saddle point in black. The colours correspond to Figure 2.

Simulations for two fields with a potential energy and initial conditions motivated by spontaneous symmetry breaking show a promising behaviour. When started from the origin, the trajectory rolls down towards the VEVs, finally settling in that with the lowest potential energy. This simulated behaviour is consistent with the fixed point analysis of the system, which can be found in Table 1. Inflation occurs near both the global and the local maxima, but the transitory evolution near the saddle point needs further detailed, preferably analytic study.

Symbol	ϕ^2	χ^2	Type
O	0	0	Repeller
P1	0	$\chi_0^2 + \frac{\lambda}{2\lambda_\chi}\phi_0^2$	Saddle
P2	$\phi_0^2 + \frac{\lambda}{2\lambda_\phi}\chi_0^2$	0	Saddle
V	ϕ_0^2	χ_0^2	Attractor

Table 1: Fixed points of the equations of motion.

2.2 Spatial evolution

With the last semester's work reproducing the literary results for a single minimally coupled scalar numerically, I turned to the case of conformal coupling and to multiple minimally coupled fields, as an intermediate step between multiple conformally coupled scalars. To this end, I first walked through the steps of regular perturbation theory with a single, minimally coupled field in order to get familiar with it, based on ref. [2].

2.2.1 Single conformally coupled scalar

With the use if the usual Hubble slow-roll parameters, the spectral index generated by a conformally coupled scalar has the literary result [3, 4] of

$$n_s - 1 \approx -2\varepsilon_1 - \varepsilon_2 + 2\varepsilon_3\,, \tag{2.3}$$

where

$$\varepsilon_3 = -\frac{ss'}{1-s^2}\,. \tag{2.4}$$

I attempted to reproduce this result, in order to prepare for the multiple scalar case. Assuming that the conserved curvature perturbation is

$$\mathcal{R} = -\frac{\delta\phi}{\overline{\phi}'} \tag{2.5}$$

Then the spectral index will have a contribution from

$$n_s - 1 \ni \frac{\mathrm{d}\ln\overline{\phi}'}{\mathrm{d}\eta} \tag{2.6}$$

To this end, the already calculated background evolution could be used. As a result I obtained the literary value, plus additional terms whose contribution is negligible, at least numerically.

2.2.2 Two minimally coupled scalars

While the analytic formulation of the two scalar case is still in progress, I aimed to test our numerical tools. To achieve this, I supposed that the spectral index for multiple scalars is the same as for one, i. e. (2.3) with $\varepsilon^{(\xi_c)} = 0$ for minimal coupling; with the only difference being the definition of the first Hubble slow roll parameter (2.2) now including multiple fields.

Through the simulated background, n_s could be evaluated at each η time point, for arbitrary initial conditions and parameters. The observed spectral tilt in the CMB then corresponds to the value picked up roughly 60 e-folds before the end of inflation. The resulting values for a typical run can be seen on Figure 2. We see four typical regions:

- Rolling down from the global maximum in blue. This is analogous to single field inflation.

- An oscillatory region around the saddle point in red. The usual treatment of perturbation theory cannot be used here, as the second slow-roll condition is violated.

- A purple region near the saddle point, when the oscillations have died off. The predicted value of n_s can be in agreement with measurements, but the existence of this patch is questionable, due to possible fine tuning artefacts.

- Rolling down from the saddle point towards the true minimum in orange. This is again analogous to single field inflation.

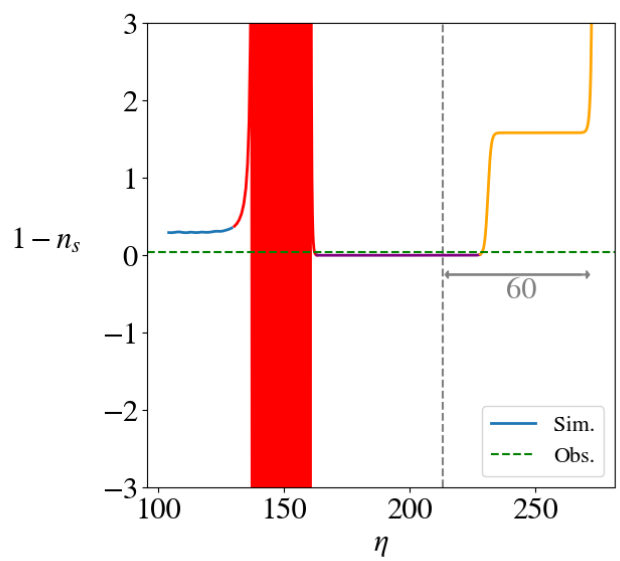

Figure 2: Spectral index along a simulated trajectory. The different colours are to separate the regions with typical behaviours.

For the calculation of the actual spectral index, I started familiarizing myself with the `xPand` package made for cosmological perturbations. The goal is to approach the results of [5], including also the scalar-scalar interaction.

3 Studies

I took three courses this semester, in preparation for the complex exam and out of personal interest in the topics covered:

- **FIZ/2/003** – Beyond the standard model
- **FIZ/2/104** – Integrable methods in gauge/gravity duality I
- **FIZ/2/138** – Experimental methods in particle physics

4 Teaching

I supervised no classes this semester, but I started preparing for the next one. In case I hold the theoretical mechanics practice class again, I will have a dedicated webpage ready for the course which will contain the administrative informations as well as my lecture notes.

5 Conferences and Grants

I participated in this years OTDK in the FiFöMa (Physics, Earth Sciences and Mathematics) section's Cosmology department. My work (titled *"Kozmológiai infláció és sötétenergia egyetlen skalárező paramétereinek függvényében"*) and presentation earned a 2nd prize.

I also applied for the EKÖP (University Research Fellowship) grant for the academic year beginning next autumn. The study is to be titled "Inflation with two, conformally coupled, symmetry broken scalar fields" (*"Infláció két, konformálisan csatolt, szimmetriasértéses skalármezővel"*), and I pledged to publish it and present it when it is done. The application is still pending as of writing this report.

References

[1] Y. Planck Collaboration: Akrami and et al. Planck2018 results: X. constraints on inflation. *Astronomy & Astrophysics*, 641:A10, September 2020.

[2] Daniel Baumann. *Cosmology*. Cambridge University Press, 2022.

[3] Valerio Faraoni. Conformally coupled inflation, 2013.

[4] Jai-chan Hwang and Hyerim Noh. Cosmological perturbations in generalized gravity theories. *Phys. Rev. D*, 54:1460–1473, 1996.

[5] Courtney M. Peterson and Max Tegmark. Testing two-field inflation. *Physical Review D*, 83(2), January 2011.