ELTE

TTK

Semester report - III.: Connecting inflation and dark energy with particle physics

Particle and Nuclear Physics program

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1 Introduction

I continued my work with scalar field cosmology, based on Lagrangians motivated by the Standard Model and its possible extensions, of the form

$$\mathcal{S} = \int \sqrt{-g} \left[\frac{1}{2} \left(M^2 - \xi \varphi^2 \right) R - \frac{1}{2} \left(D\varphi \right)^2 + V(\varphi) \right] \mathrm{d}^4 x \,. \tag{1.1}$$

We allow the abstract φ to represent an arbitrary multiplet of complex scalar fields, but restrict the scalar-curvature coupling ξ to either the minimal $\xi = 0$ or conformal $\xi = 1/6$ value. From the equations of motion the evolution can be obtained and related to observations, as mentioned in previous semester reports.

I've derived the exact equations of motion for both the minimally and conformally coupled cases, optionally with an ideal fluid permeating the universe, for an arbitrary number of scalar fields, and for an arbitrary potential energy. I've found approximate analytical solutions for the inflationary era, which match my exact numerical simulations under some assumptions. For later epochs, the solution is in progress.

With the spectral parameters of a single minimally coupled field estimated, I turned to the more complicated cases. Having found literary results for the perturbations of a single conformally coupled scalar, I set out to check and compare them to my calculations. With confomal transformation techniques, I could relate them to my minimally coupled simulations, and check the level of numerical agreement They do not agree exactly, but the difference is negligible at present precision of measurements. With this done, the next step is to generalize the perturbation theory to multiple conformally coupled scalars.

2 Guided research

2.1 Temporal evolution

2.1.1 The general equations

Starting from the Lagrangian in (1.1), one can derive variationally the Klein-Gordon and Einstein equations, the latter of which lead to the Friedmann equations under the assumption of a (flat) FLRW metric. It is also notable that the terms involving the scalar separate, therefore an N multiplet of complex scalars effectively reduces to $2N$ real scalar fields that only couple to each other through the potential. Further energy constituents, like an ideal fluid acting either as matter ($w_f = 0$) or radiation ($w_f = 1/3$) can readily be added, as can an effective decay between the scalar(s) and these constituents.

For example, the complete system of equations for minimally coupled scalars reads

$$\underline{S}'' = -\frac{3}{2}\Big[(1-w_f)\big(1-\Omega_S\big) + 2(\Omega_S - \underline{S}'^2)\Big]\underline{S}' - \frac{1}{2}(\Omega_S - \underline{S}'^2)\partial_{\underline{S}} \ln \mathcal{V}(\underline{S}) + \underline{\underline{\tilde{\Gamma}}}\, \underline{S}' \,, \tag{2.1}$$

$$\Omega_S' = \big(1-\Omega_S\big)\big[3(1+w_f)\Omega_S - 6\underline{S}'^2\big] + 2\underline{\underline{\tilde{\Gamma}}}\, \underline{S}'^2 \,, \tag{2.2}$$

where the prime denotes differentiation with respect to e-fold η, $\underline{S}$ is a vector containing all (rescaled) scalar fields, Ω_s is the relative energy density contained in the fields, and $\underline{\underline{\tilde{\Gamma}}}\,'\underline{S}$ is an effective decay term that describes how the fields decay into radiation or matter. The same kind of equation can be derived for conformal ($\xi = 1/6$) coupling as well; and restricting our studies to inflation only corresponds to setting $\Omega_s = 1$ and $\underline{\underline{\tilde{\Gamma}}} = 0$.

2.1.2 An analytic solution

During the inflationary era, if the potential can be approximated in such a way that

$$-\frac{1}{2}\partial_{\underline{S}} \ln \mathcal{V}(\underline{S}) \approx \underline{\underline{A}}\,\underline{S} \tag{2.3}$$

and $\underline{S}'^2 \ll 1$, then equation (2.1) reduces to the form

$$\underline{S}'' \approx -3\underline{S}' + \underline{\underline{A}}\,\underline{S}\,. \tag{2.4}$$

If $\underline{\underline{A}}$ is diagonal then this equation is solved by exponential functions of the e-folds η. We show an example of this solution compared to the simulations on Figure 1a, for the single field case assuming a BEH-like potential with a vacuum expectation value s_0.

Our quantities of interest in perturbation theory lie 50–60 e-folds before the end of inflation η_0. Therefore a further use of this approximation is to notice that these quantities do not depend on the initial conditions from which our system starts (as long as it is near the origin). We find that for all initial conditions the variables will converge to the common solution, as can be seen on 1b. Hence as long as we have at least 50–60 e-folds, then the model has no fine-tuning problem, and observables are independent of the precise value of initial conditions.

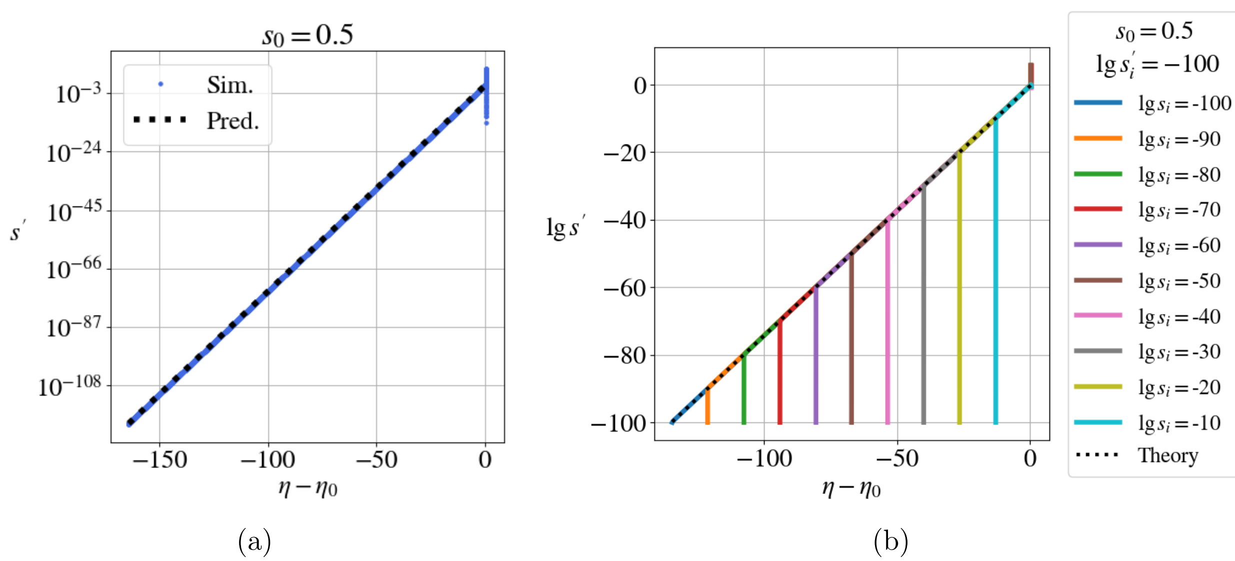

Figure 1: (a) Comparison of the analytic prediction and the simulations for a sample parameter s_0, the vacuum expectation value of the field in units of Planck mass. The fields derivative is shown as a function of the e-folds η shifted so that the end of inflation occurs at some η_0 value. (b) Comparison of trajectories starting from different initial conditions. We see that they all approach the common line given by the analytic approximation.

We note that while the above equations and plots are for the minimally coupled case, the same has been done for the conformal one as well. The scenario with multiple fields is in progress, but the numerical results seem consistent with what one would expect.

2.2 Spatial evolution

As mentioned in previous reports, the spectral parameters can be related to the background simulated above. This is now complete for the single minimally coupled scalar, as can be seen on Figure 2.

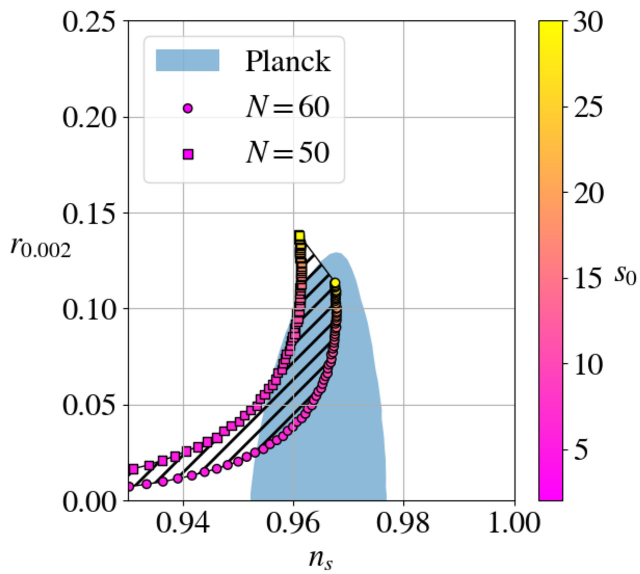

Figure 2: The spectral parameters n_s (the spectral index of scalar perturbations) and r (the scalar-tensor ratio of the perturbation amplitudes) predicted by the minimally coupled single BEH field model. The values were evaluated $N = (50, 60)$ e-folds before the end of inflation. With blue we denoted the observationally permitted region based on [1].

For conformal coupling the observables are less trivial. With the help of literary results [2] I could reach a possible form of equations that can be evaluated. The idea is to use the conformal map between minimally and conformally coupled Lagrangians to relate the perturbations between these so-called frames.

This can be done at two levels. Applying the map to the Lagrangian itself leads to a redefined field, with a new potential energy

$$V(s) \propto (s^2 - s_0^2)^2 \qquad \longrightarrow \qquad V(\hat{s}) \propto \frac{(\tanh^2 \hat{s} - \tanh^2 \hat{s}_0)^2}{(1 - \tanh^2 \hat{s})^2}, \tag{2.5}$$

from which we obtain a minimally coupled model with a modified potential. This can be evaluated as usual. On the other hand, the inverse map can be applied to the curvature perturbations one obtains with minimal coupling; leading to the relation between them and the simulated field of the conformally coupled Lagrangian. In theory, the spectral parameters calculated in these two ways should overlap. In practice, they differ slightly, the reason for which is not yet understood.

3 Studies

I took and completed two courses this semester:

- **FIZ/2/084E** – Integrable field theories
- **FIZ/2/004E** – Experimental methods of particle physics II.

4 Teaching

I supervised the following practice class again:

- **elmfiz1af17ga** – Elméleti mechanika A *(gyakorlat)*

Which accounts to 90 minutes of teaching each week, to the 74 students who took this course. Due to the large number, I also had some much appreciated help in grading. Compared to last year, my lecture notes were refined; and a dedicated webpage made for students to access both them, and the administrative information about the course.

5 Conferences and Grants

I've applied to a spring school organized by the Galileo Galilei Institute in Firenze, titled *Theoretical Aspects of Astroparticle Physics, Cosmology and Gravitation*, to be held between March 2 and March 13. I got accepted, and intend to participate.

References

[1] Y. Planck Collaboration: Akrami and et al. Planck2018 results: X. constraints on inflation. *Astronomy & Astrophysics*, 641:A10, September 2020.

[2] José Jaime Terente Díaz and Mindaugas Karčiauskas. Comoving curvature perturbation in jordan and einstein frames. *Physical Review D*, 108(8), October 2023.