IV. Semester report

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Ph.D. Thesis title: Consequences of thermodynamic constraints in the classical theory of gravitation
(Termodinamikai kényszerek következményei a klasszikus gravitációelméletben)

2024/2025 II. semester

[1]HUN-REN Wigner RCP

Introduction

My planned research applies extensions of modern continuum theories to produce astrophysically relevant predictions. In the investigated theory [1], the second law of thermodynamics is used as a constraint in a nonequilibrium framework. The derived evolution equation with an introduced scalar field yields Poisson's equation of Newtonian gravity, or in an extended version, its modified form. The extension results in nonlinear field equation, which is tested against astrophysical phenomena, specifically against galactic rotation curves. This method allows the explanation of some dark matter-related phenomena without introducing exotic, new matter or particles. Modified gravity has been researched for decades to explain cosmological phenomena and issues with the dark matter model, but in my research, a novel and unique approach is tested. Furthermore, its potential applications for the explanation of non-galactic phenomena is considered an open issue, which warrants further research.

Research work in semesters I-III.

I. semester

In this semester, I further developed the previously created algorithm to introduce numerical fitting to the observed galactic rotation curves. The method takes the difference between the resulting numerical rotation curve and the observational data and fits the parameter K (with optionally the outer boundary condition as well), using the least squares method of the curve_fit function of the Scipy.optimize python module. The result yields a better-aligned rotation curve and an error estimation from the fit for its value. One such result is shown in Fig. 1.

Furthermore, the modelling technique for the galaxies was further developed. A more befitting coordinate system is the cylindrical one, compared to the previously used spherical, thus in cases where a compatible density distribution could be tested, the more appropriate former was used. This allowed me to test the larger number of galaxies in [2], in the form of coadded rotation curves. The density distribution here was an infinitely thin disk, characterised by its mass and exponential length scale. One such example is shown in Fig. 2.

Further research is underway in investigating a potentially more appropriate numerical method, by using a more suited cylindrical or spherical derivative of $r\nabla\varphi$ and $r^2\nabla\varphi$ instead of $\nabla\varphi$ in the staggered grid method.

II. semester

In this semester, based on reviewer feedback, I incorporated the SPARC (Spitzer Photometry & Accurate Rotation Curves) database to work with the previously created numerical method of applying thermodynamic gravity to galactic rotation curves. To enhance the quality of the method, I enhanced the developed a numerical fitting method in order to calculate the best fits and a reduced chi-squared value. In order to account for the observational uncertainties and improve the fit, I let the mass-to-light ratio of the galactic disc vary as a free parameter, while constraining the outer boundary to the observed value. Using this new method, I improved and resubmitted the manuscript mentioned below. An example of this new method applied to the galaxy discussed in our work is presented in Figure 3.

Furthermore, the planetary precession was investigated by using GravityLab (`https://pypi.org/project/GravityLab/`). The numerical simulation showed that under similar starting conditions, thermodynamic gravity results in precession, as seen in Fig. 4. This research avenue, if further investigated, could potentially yield Solar system-regime tests of thermodynamic gravity, or predictions about the gravitational stability of planetary systems.

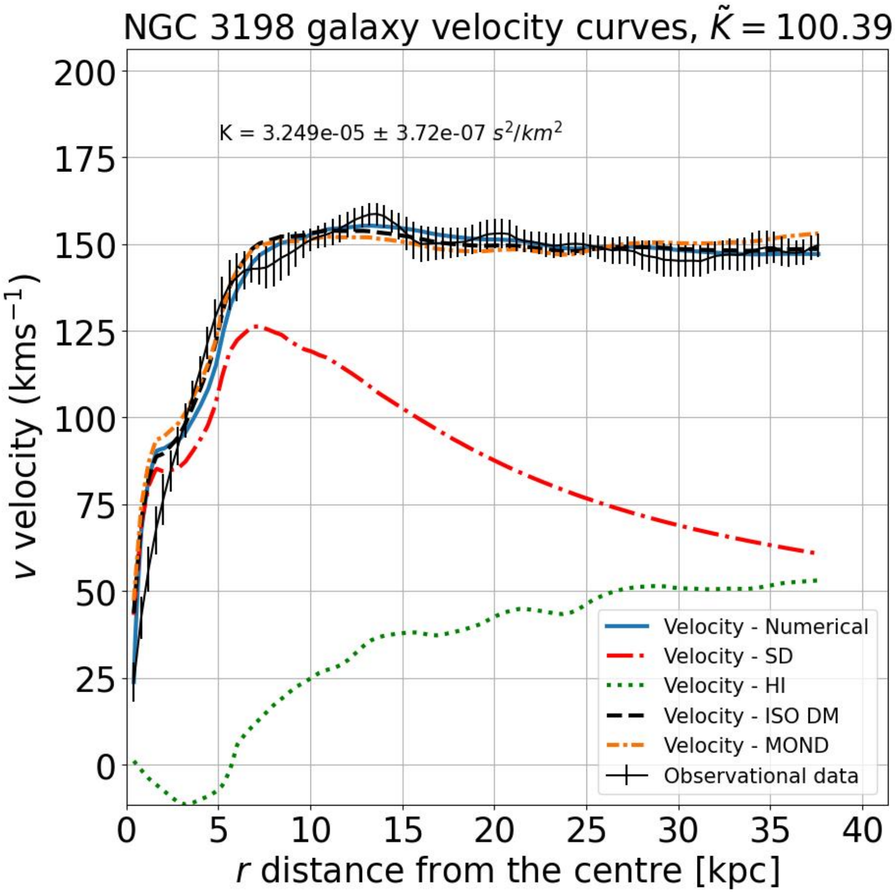

Figure 1: Numerical fitting for the galaxy NGC 3198, with K and the outer boundary condition being a free parameter.

Further research is underway to produce fits for the rest of the SPARC sample, considering the uncertainties in the observed inclinations and distances of the galaxies. If a large enough sample is investigated, Bayesian analysis should provide a potential way to compare the model's success with other, more developed mainstream models, such as various dark matter frameworks or the external field effect realisation of MOND (Modified Newtonian Dynamics). Additionally, developing a proper numerical method for cylindrical symmetric systems, to improve the fits of the [2] sample, while investigating enhancements to the applied staggered grid method.

III. semester

In this semester, I further investigated the potential Solar system-level predictions of thermodynamic gravity, by calculating the difference between the Newtonian and TG predictions of the perihelion precession of Mercury and light bending around the Sun. For the latter, in the simple Newtonian model, an analytical solution for the angular deflection Θ of a light particle as it travels near a star with mass M, approaching it at a distance of R at the closest point:

$$\Theta_{Newtonian} = \frac{2GM}{c^2R}, \tag{1}$$

$$\Theta_{TG} = \frac{2GM}{c^2R} \cdot \frac{R}{KGM} \cdot \left[\frac{\pi}{2}\left(1 - \frac{R}{\sqrt{R^2 - G^2M^2K^2}}\right) + \frac{R}{\sqrt{R^2 - G^2M^2K^2}} \cdot \arctan\left(\frac{GMK}{\sqrt{R^2 - G^2M^2K^2}}\right)\right]. \tag{2}$$

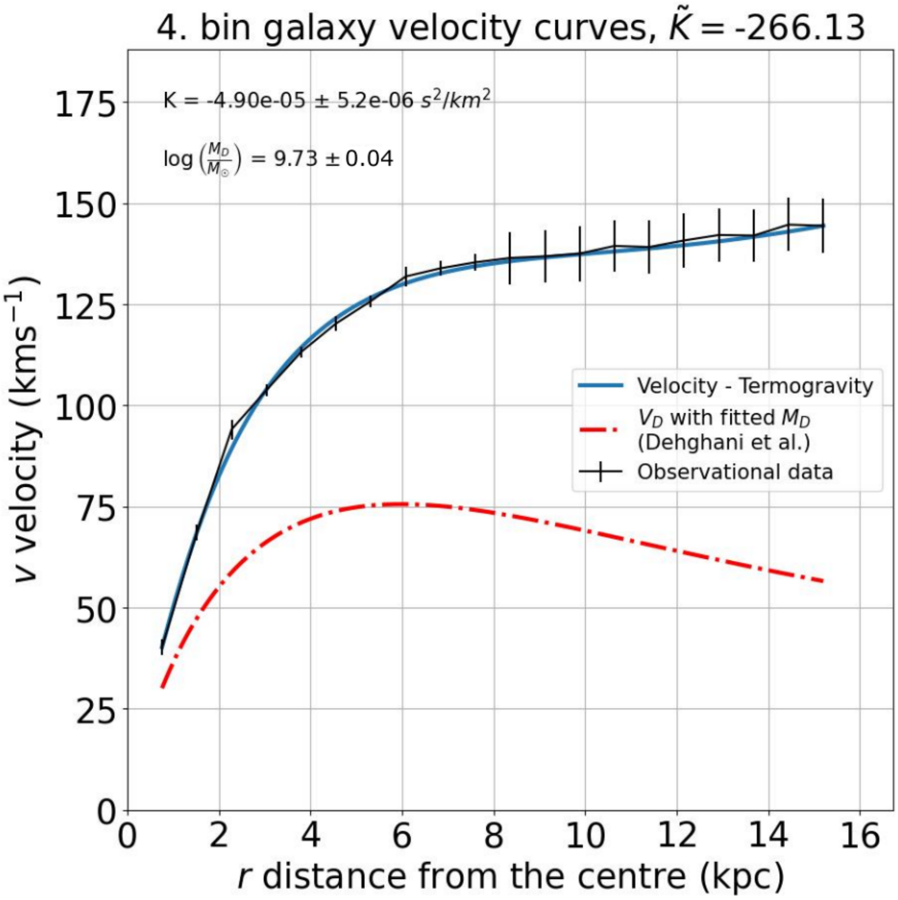

Figure 2: Numerical fitting for the coadded rotation curves of bin 4 from [2], with the mass of the exponential disk (M_D) and the parameter K being a free parameter.

An important research avenue this semester was the incorporation and comparison of the self-consistent gravity approach with TG. Self-consistent gravity [3] accounts for the relativistic effect of the gravitational field's energy being equivalent with mass, per SR, and produces a modified Poisson's equation to account for this:

$$\Delta\varphi = \frac{4\pi G}{c^2}\rho\varphi + \frac{1}{2\varphi}(\nabla\varphi)^2. \tag{3}$$

By using a general function for thermodynamic gravity with heuristic approach to derive TG, we can use a different self-energy term instead of the one in the usual derivation. Thus, from the general initial equation of

$$u = e - \varphi - \frac{\Sigma(\varphi, \nabla\varphi)}{\rho}, \tag{4}$$

we can obtain in the case of $\Sigma(\varphi, \nabla\varphi) = \frac{c^2}{8\pi G\varphi}(\nabla\varphi)^2$ (to account for the effects of the self-consistent corrections):

$$\Delta\varphi = \frac{1}{2}\left(\frac{1}{\varphi} + 2K\right)(\nabla\varphi)^2 + \frac{4\pi G}{c^2}\rho\varphi, \tag{5}$$

as is Eq. 22 in [3] in the $K = 0$ limit. By writing the equation with $\sqrt{\varphi}$, we can get the following form:

$$\Delta(\sqrt{\varphi}) = \frac{2\pi G}{c^2}\rho\sqrt{\varphi} + 2K\sqrt{\varphi}\left(\nabla\left(\sqrt{\varphi}\right)\right)^2. \tag{6}$$

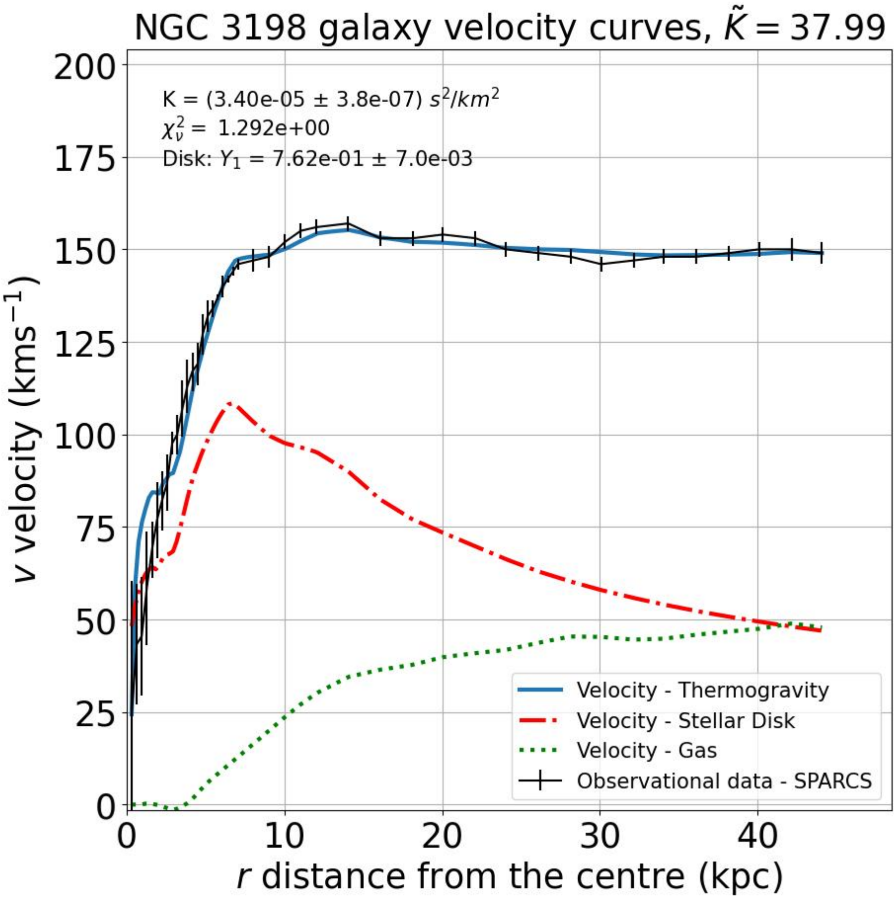

Figure 3: Numerical fitting for the galaxy NGC 3198, with K and the mass-luminosity ratio (Y_1) being a free parameter.

In the case of vacuum ($\rho(r) = 0$), the solution is

$$\varphi(r) = -\frac{1}{K}\mathrm{erf}^{-1}\left(-\sqrt{-\frac{K}{c^2\pi}}\left(2c^2 - \frac{GM}{r}\right)\right)^2. \tag{7}$$

I implemented a numerical solution to solve the case of a constant-density region conjoined to an outer vacuum (Fig. 5):

I also calculated the potential effects of including the cross-effect coupling term from TG within the method described in [3], which resulted in the following form of modified Poisson's equation:

$$\Delta\varphi = K(\nabla\varphi)^2\frac{1/2 - e^{K\varphi}}{1 - e^{K\varphi}}, \tag{8}$$

but solutions consistent with the usual self-consistent results in the $K = 0$ limit require further research.

A research avenue to produce fits for the rest of the SPARC sample is planned via the traditional TG method with the potential inclusion of insights from self-consistent gravitational solutions. Fitting the inclinations and distances of the galaxies may produce better results, and if a large enough sample is investigated, Bayesian analysis could provide a potential way to compare the model's success with other, more developed mainstream models.

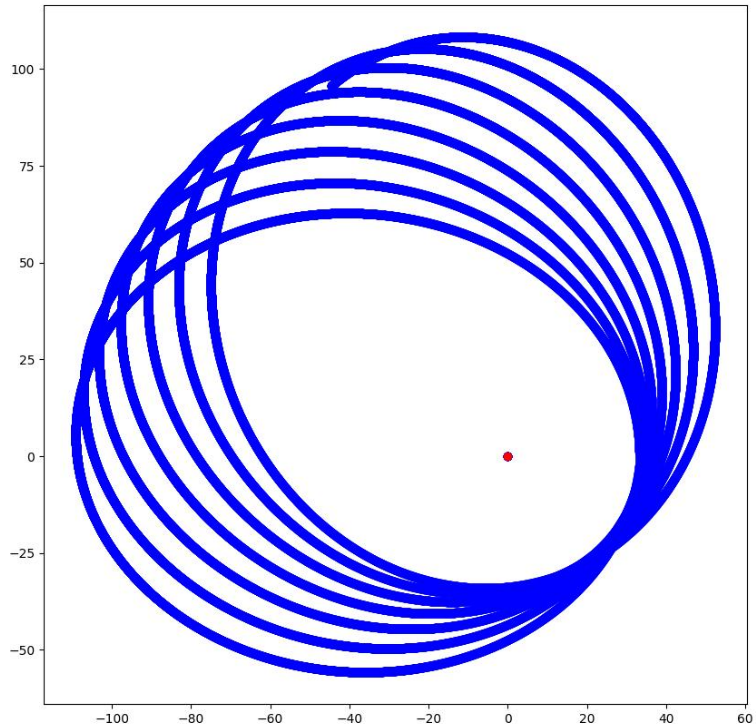

Figure 4: A simulation with bodies reflecting the Sun-Earth mass ratio, with $K = 10^{-5}$ (relative to other the simulation parameters) TG parameter.

Research work in semester IV.

In this semester, I further developed the general, higher-gradient dependent function derivation as a part of the internal energy, in the following form:

$$u = e - \frac{\varepsilon(\rho, \varphi, \nabla\varphi, \nabla^2\varphi, \nabla^3\varphi)}{\rho}, \tag{9}$$

where a scalar field φ and its gradient $\nabla\varphi$ plus its higher gradients $\nabla\otimes\nabla\varphi = \nabla^2\varphi$ and $\nabla\otimes\nabla\otimes\nabla\varphi = \nabla^3\varphi$ are part of the state space. For the full thermogravitational potential, the following field equation was obtained, analogous to the method used in earlier research:

$$\begin{aligned} \tau\partial_t\varphi = l^2 4\pi G \Bigg(& \nabla\cdot(\partial_{\nabla\varphi}\varepsilon) - \partial_\varphi\varepsilon - \nabla^2 : \partial_{\nabla^2\varphi}\varepsilon + \nabla^3 \vdots \partial_{\nabla^3\varphi}\varepsilon + \\ & + 2K\Big[-3\varepsilon + 3\rho\partial_\rho\varepsilon + (\partial_{\nabla\varphi}\varepsilon - \nabla\cdot\partial_{\nabla^2\varphi}\varepsilon + \nabla^2 : \partial_{\nabla^3\varphi}\varepsilon)\cdot(\nabla\varphi) \\ & + (\partial_{\nabla^2\varphi}\varepsilon - \nabla\cdot\partial_{\nabla^3\varphi}\varepsilon) : (\nabla^2\varphi) + (\partial_{\nabla^3\varphi}\varepsilon)\vdots(\nabla^3\varphi)\Big]\Bigg). \end{aligned} \tag{10}$$

It can also be shown that the classical holographic property holds for this method involving the general function. The classical holographic property for an arbitrary field ϕ in thermodynamics can be summarised as

$$\nabla\cdot\mathbf{P}(\chi) = \rho\nabla\phi(\chi), \tag{11}$$

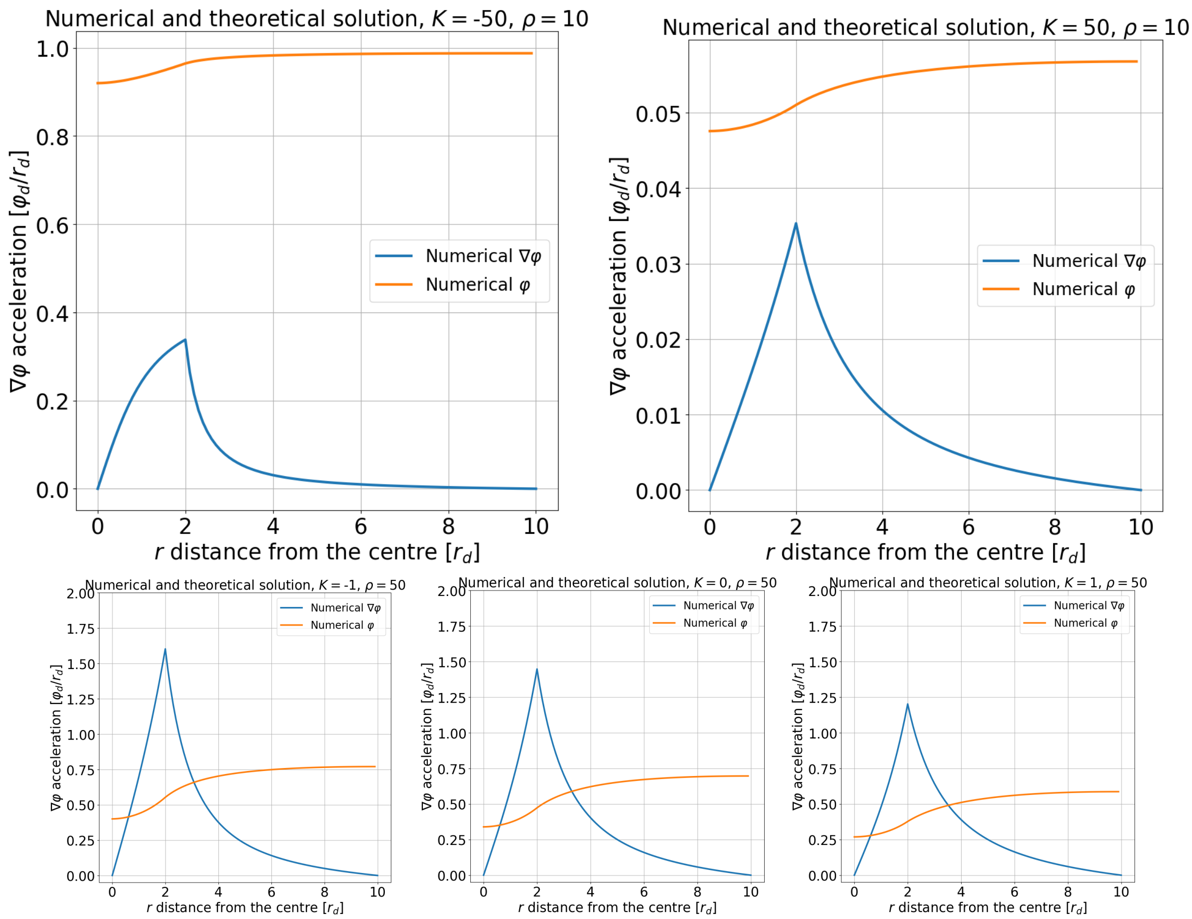

Figure 5: The result of the numerical method compared to the theoretical solutions with various K and ρ values. Constant density up to $r = 2r_d$, then vacuum. Boundary conditions $\nabla\varphi(0) = \nabla\varphi(10r_d) = 0$.

where χ denotes the constitutive state space. In our case, from the mechanical thermodynamic flux and the constraint by the II. law, the following equation can be written:

$$\mathbf{P} + (\varepsilon - p - \rho\partial_\rho\varepsilon)\mathbf{I} - (\partial_{\nabla\varphi}\varepsilon - \nabla\cdot\partial_{\nabla^2\varphi}\varepsilon + \nabla^2 : \partial_{\nabla^3\varphi}\varepsilon)(\nabla\varphi)$$
$$-(\partial_{\nabla^2\varphi}\varepsilon - \nabla\cdot\partial_{\nabla^3\varphi}\varepsilon)\cdot(\nabla^2\varphi) - (\partial_{\nabla^3\varphi}\varepsilon) : (\nabla^3\varphi) = 0. \tag{12}$$

Additionally, using the Gibbs-Duhem relation,

$$0 = s\mathrm{d}T - v\mathrm{d}p + \mathrm{d}\mu, \tag{13}$$

to write

$$\nabla p = \rho\nabla\mu + \rho s\nabla T, \tag{14}$$

we can write the following:

$$\nabla\cdot\mathbf{P} = -\nabla\varepsilon + \rho\nabla\mu + \rho s\nabla T + \rho\nabla\partial_\rho\varepsilon + \partial_\rho\varepsilon\nabla\rho + (\nabla\cdot\partial_{\nabla\varphi}\varepsilon - \nabla^2 : \partial_{\nabla^2\varphi}\varepsilon + \nabla^3 \vdots\, \partial_{\nabla^3\varphi}\varepsilon)(\nabla\varphi)$$
$$+(\partial_{\nabla\varphi}\varepsilon - \nabla\cdot\partial_{\nabla^2\varphi}\varepsilon + \nabla^2 : \partial_{\nabla^3\varphi}\varepsilon)\cdot(\nabla^2\varphi) + (\nabla\cdot\partial_{\nabla^2\varphi}\varepsilon - \nabla^2 : \partial_{\nabla^3\varphi}\varepsilon)\cdot(\nabla^2\varphi) + (\partial_{\nabla^2\varphi}\varepsilon - \nabla\cdot\partial_{\nabla^3\varphi}\varepsilon) : (\nabla^3\varphi)$$
$$+(\nabla\cdot\partial_{\nabla^3\varphi}\varepsilon) : (\nabla^3\varphi) + (\partial_{\nabla^3\varphi}\varepsilon)\,\vdots\,(\nabla^4\varphi). \tag{15}$$

Since it is straightforward to expand $\nabla\varepsilon$,

$$\nabla\varepsilon = \partial_\rho\varepsilon\nabla\rho + \partial_\varphi\varepsilon\nabla\varphi + \partial_{\nabla\varphi}\varepsilon\cdot\nabla^2\varphi + \partial_{\nabla^2\varphi}\varepsilon : \nabla^3\varphi + \partial_{\nabla^3\varphi}\varepsilon \vdots \nabla^4\varphi, \tag{16}$$

and using the constraint of the gravitational thermodynamic force with the II. law,

$$\nabla \cdot \partial_{\nabla\varphi}\varepsilon - \partial_{\varphi}\varepsilon - \nabla^2 : \partial_{\nabla^2\varphi}\varepsilon + \nabla^3 \vdots \partial_{\nabla^3\varphi}\varepsilon = 0, \tag{17}$$

we can simplify eq. 15 to obtain:

$$\nabla \cdot \mathbf{P} = \rho\nabla\mu + \rho s \nabla T + \rho\nabla\partial_{\rho}\varepsilon. \tag{18}$$

We can see that the classical holographic property does apply here.

Using this general function, aside from the previously investigated self-consistent and standard thermogravitational cases, I also considered the following $\varepsilon(\varphi, \nabla\varphi, \nabla^2\varphi, \nabla^3\varphi)$ from M. Lazar [4], eqs. 3-4.:

$$\mathscr{L} = \frac{1}{8\pi G}\left((\nabla\varphi)^2 + l_1^2\left(\nabla^2\varphi\right)^2 + l_2^4\left(\nabla^3\varphi\right)^2\right) + \rho\varphi = \varepsilon, \tag{19}$$

This yields from eq. 10 the following:

$$\tau\partial_t\varphi = l^2 4\pi G\Bigg(\frac{1}{4\pi G}\Delta\varphi - \rho - \frac{l_1^2}{4\pi G}\Delta^2\varphi + \frac{l_2^4}{4\pi G}\Delta^3\varphi$$

$$-\frac{K}{4\pi G}\left((\nabla\varphi)^2 + l_1^2\left(\nabla^2\varphi\right)^2 + l_2^4\left(\nabla^3\varphi\right)^2 + l_1^2\Delta(\nabla\varphi)^2 + l_2^4\Delta\left[(\nabla^2\varphi)^2 - \Delta(\nabla\varphi)^2\right]\right)\Bigg). \tag{20}$$

In the stationary solution, this simplifies to:

$$4\pi G\rho = \Delta\varphi - l_1^2\Delta^2\varphi + l_2^4\Delta^3\varphi$$

$$-K\left((\nabla\varphi)^2 + l_1^2\left[\left(\nabla^2\varphi\right)^2 + \Delta(\nabla\varphi)^2\right] + l_2^4\left[\left(\nabla^3\varphi\right)^2 + \Delta(\nabla^2\varphi)^2 - \Delta^2(\nabla\varphi)^2\right]\right), \tag{21}$$

which reduces to the form in [4], in the $K = 0$ case.

Publications

Publications in the 1st semester

- Published science communication article:

  P. Ván, M. Pszota: Equivalence principles and gravitation theories, Fizikai Szemle, December 2023
  (`https://fizikaiszemle.elft.hu/szemle/123`)

Publications in the 2nd semester

- Published science communication article:

  G. Balázs, K. Bánhalmi, B. Creusot, K. Császár, E. Gasteiger, Á. Horti-Dávid, P. Horváth, M. Kovács, N. Koródi, S. Marek, C. Porpaczy, M. Pszota: A korai univerzum vörös lámpása: hiányzó láncszem lehet a távoli kvazár, csillagaszat.hu, 19th May 2024
  (`https://www.csillagaszat.hu/hirek/a-korai-univerzum-voros-lampasa-hianyzo-lancsz`

Publications in the 3rd semester

- Published article:

  M. Pszota and P. Ván. *Field equation of thermodynamic gravity and galactic rotational curves.* Physics of the Dark Universe, 46:101660, 2024.

Publications in the 4th semester

- Manuscript under preparation (draft phase):

  M. Pszota and P. Ván. *Thermodynamically extended theories of modified gravity*

Studies in the 4th semester

- Galactic Dynamics (Csillagrendszerek dinamikája I.) (FIZ/5/027)

  This course aims to explore the basics of gravitational dynamics and its application to systems ranging from planetary and stellar systems to clusters of galaxies. Starting from two-body motion and orbits under a central-force law, the course investigates the dynamics of binary systems and then goes on to explore many body systems from stellar clusters to galaxies. Methods of determining gravitational potentials and orbits in these potentials are explored. The course then develops the statistical treatment of stellar motions including two-body relaxation and the use of the collisionless Boltzmann equation with applications to observations of stellar systems.

- Gravitational-wave astrophysics (Gravitációshullám-asztrofizika) (FIZ/5/021)

  During this course, we received an introduction to gravitational waves (GW) and the basic design properties of GW detectors. We explored the mathematical machinery necessary to describe this curvature and then briefly discussed the formulation and solution of Einstein's field equations, which encode the relationship between the curvature of spacetime and its matter and energy content. We also learned about the properties of the GW detectors, data representation and potential noise factors. Finally, we sampled literature about the cosmological use of GW signals to determine the Hubble parameter.

Conferences

I. semester

- Participation at the Wigner 121 Scientific Symposium, Budapest, Hungary, 18-20 September 2023

- Oral presentation at the Zimányi School 2023 (23rd Zimányi School Winter Workshop on Heavy Ion Physics) with the title *Galactic hydrodynamics with thermodynamic gravity*, Budapest, Hungary (`http://zimanyischool.kfki.hu/23/`), December 4-8, 2023

II. semester

- Participation at the XII Bolyai–Gauss–Lobachevsky Conference (BGL-2024): Non-Euclidean Geometry in Modern Physics and Mathematics, Budapest, Hungary, 1-3 May 2024

- Participation at the Exploring the Dark Side of the Universe Tools 2024 - 5th World Summit, Île de Noirmoutier, France, 2-7 June 2024

III. semester

- Oral presentation at the International Conference on Thermodynamics 2.0 (online) with the title *Gravity in non-equilibrium Thermodynamics*, Appalachian State University, Boone, USA, August 05-07, 2024, (`https://iaisae.org/index.php/agenda-schedule-t2022/`).
- Oral presentation at Time Machine Factory [unspeakable, speakable] on Time Travel - 2024 with the title *Gravity and nonequilibrium thermodynamics: the origin of evolution equations*, Turin, Italy, September 22-25, 2024, (`https://indico.ict.inaf.it/event/751/overview`).
- Oral presentation at Simonyi-Nap 2024 with the title *A sötét Univerzum - a gravitáció módosítása a kulcs az évtizedes rejtélyhez?*, Budapest, Hungary, October 10th, 2024, (`https://wigner.hu/hu/simonyi-nap-2024-0`).
- Oral presentation at Statisztikus Fizikai Nap 2024 with the title *Termodinamika, gravitáció és asztrofizikai alkalmazása*, Budapest, Hungary, October 25th, 2024, (`https://sites.google.com/view/statfiznap24/`).
- Poster presentation at Zimányi School Winter Workshop 2024 with the title *Galactic rotation curves with thermodynamic gravity*, Budapest, Hungary, December 02-06, 2024, (`https://zimanyischool.kfki.hu/24/`).

IV. semester

- Oral presentation at the Joint European Thermodynamics Conference with the title *Derivation of higher order gradient thermodynamic gravity*, Belgrade, Serbia, May 26-30, 2025, (`https://www.mi.sanu.ac.rs/JETC2025/index.html`)

Further activities

I. semester

Teaching activity

I participated in teaching the Classical Physics Laboratory (laboratory practice), for 10x4 hours (once per week 4 hours, for a total of 40 hours).

Science communication

I participated in the organisation of the Náboj physics contest for secondary school students on 3rd November 2023 and in the Night of Researchers demonstrations at the ELTE on 29th September 2023.

II. semester

Teaching activity

I participated in teaching the Classical Physics Laboratory (laboratory practice), for 10x4 hours (once per week 4 hours, for a total of 40 hours).

III. semester

Teaching activity

I participated in teaching the Classical Physics Laboratory (laboratory practice), for 10x4 hours (once per week 4 hours, for a total of 40 hours).

Science communication

I participated in the organisation of the Náboj physics contest for secondary school students on 15th November 2024 and in the Night of Researchers demonstrations at the ELTE and HUN-REN Wigner RCP on 27th September 2024.

IV. semester

Teaching activity

I participated in teaching the Classical Physics Laboratory (laboratory practice), for 10x4 hours (once per week 4 hours, for a total of 40 hours).

References

[1] Peter Ván and Sumiyoshi Abe. Emergence of extended Newtonian gravity from thermodynamics. *Physica A: Statistical Mechanics and its Applications*, 588:126505, 2022.

[2] Dehghani, R., Salucci, P., and Ghaffarnejad, H. Navarro-Frenk-White dark matter profile and the dark halos around disk systems. A&A, 643:A161, 2020.

[3] J. Franklin. Self-consistent, self-coupled scalar gravity. *American Journal of Physics*, 83(4):332–337, 04 2015.

[4] Markus Lazar. Gradient modification of Newtonian gravity. *Phys. Rev. D*, 102:096002, Nov 2020.