Semester Report

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PhD Program: Statistical physics

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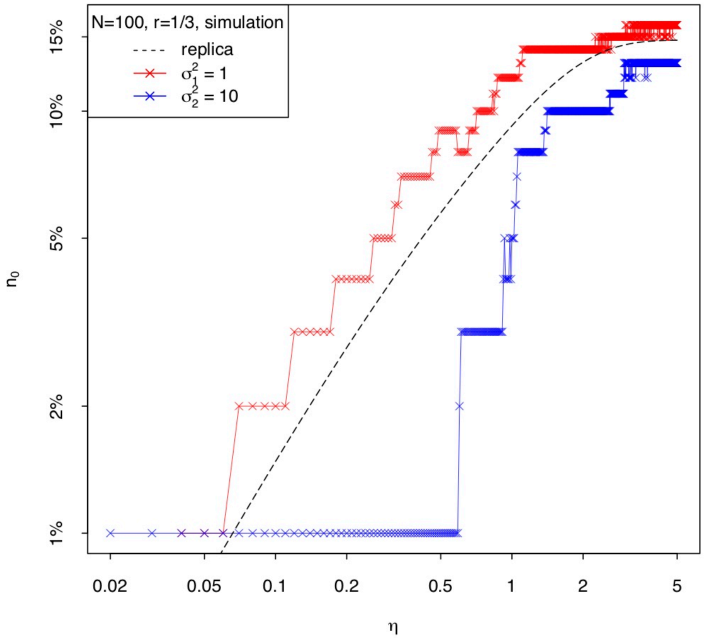
Introduction:

Portfolio optimization is a formal mathematical approach to making investment decisions across a collection of financial instruments or assets. Variance optimization under Expected Shortfall, L_1 and L_2 constraints are carried out analytically by replica method from statistical physics. The replica methods are effectively to study the sensitivity to estimation error of portfolios in the high dimension. By considering portfolios of varying large sizes N and for different lengths T of the time series with their ratio $r = N/T$ finite, the noise has a very strong effect in all investigated cases, asymptotically it only depends on the N/T ratio and becomes infinity at a critical value N/T. Obviously, this divergence is the manifestation of a phase transition, and this transition is accompanied by a number of critical phenomena, including the divergent sample to sample fluctuations of portfolio weights. By construct the lines along which the estimation error takes up finite, a contour map of the estimation error for ES are presented by *F. Caccioli et al. 2017*. Optimization with no short-selling (*I. Kondor et al. 2017*) and L_1 regularization (*Kondor et al. 2017*) are also be discussed in replica method. Now, the structure of covariance matrix in objective function of optimization is considered to study the behavior of portfolio weights and critical phenomena, which is my present work.

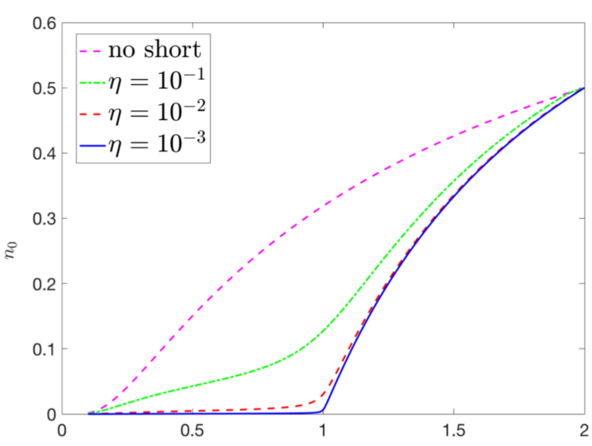
Description of research work carried out in current semester:

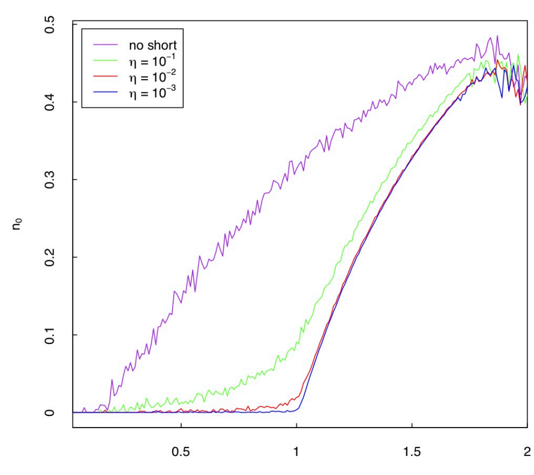
At the beginning, undoubtable I have to learn the replica method in optimization from current papers. By focusing on the analytic approach to variance optimization under an L_1 constraint, I repeat the derivation of analytical part from effective objective function with budget constraint to the analytical expression of weight distribution step by step. The L_1 regularizer is defined asymmetrically, having different slopes for positive and negative weights. The penalty term of each kind of weights are η_1 and η_2 and a special case $\eta_1 = \eta_2 = \eta$ corresponds to the usual expression, Keeping the two slopes different allows us to penalize long and short positions differently. In general, L_1 regularizer eliminates some of the assets from the portfolio by setting their weight to zero. And the expression of weight distribution from replica method also shows an item of zero weight n_0. To compare the relation between regularization parameter η and n_0, a simulation method of *lasso*(least absolute shrinkage and selection operator) is considered, which is known to eliminate some of the variable with zero weights. By "*quadprog*" package in R language, the weights and relative results of optimization under L_1 regularizer with random



samples of Normal distribution in simulation method variables is carried out. Then replica method is also set the same data as a comparison with simulation result.

In the above figure, numerical results for a two-variance portfolio and compare them to the results of the replica calculation. The numerical model is constructed from $N = 100$ assets, each having $T = 300$ data points ($r = 1/3$) drawn from a normal distribution. The variance of the returns is set to be $\sigma_2 = 1$ for half of the assets, while the other half has $\sigma_1 = \sqrt{10}$. The proportion of zero weights n_0 as function of eta for two different single samples (blue and red) of a portfolio with the same composition of 100 assets. The step-like functions are more or less following the trend of the theoretical curve, which has been derived in the large N limit and shown in the figure by a black dashed line, but the fluctuations for $N = 100$ are still large.





With L_1 regularizer, the range of r extends from $0 < r < 1$ to $0 < r < 2$. Now we carry out the results of zero weights n_0 as a function of r with different coefficient eta in replica and numerical method. For simplicity, we can henceforth set $\sigma_1 = \sigma_i = 1$ for all i. The left figure is the fraction n_0 of zero weights as function of r in replica method from *I. Kondor et al. 2017* and the right one is my simulation approach. We can see that for small values of the coefficient η of the regularizer n_0 is very small for $r < 1$, but starts increasing fast above $r = 1$, ultimately going to 1/2. It is clearly that there is a sharp fluctuation when r is close to 2 by simulation but directly a smooth curve to $n_0 = 0.5$ by replica, which shows a little difference between the two methods.

The further work is on the relation between zero weights n_0 and variance matrix σ of different structure. We have considered σ as a diagonal matrix, and now set it as a symmetric matrix to study the effect of variance to n_0.

Studies in current semester:

ELTE courses:

FIZ/KUT-S1A	Guided research work	(Dr. Gábor Papp)
FIZ/3/032E	Phase transition	(Dr. Sasvári László)
BMVD-065:1	General Hungarian Language Course I.	(Ms. Polgár Emese)

Conferences in current semester:

Statistical Physical seminar at ELTE Institute of Physics:

“Generalized quantum Zeno dynamics and ergodic means”, Zoltán Zimborás, Oct 24.

“Network-theory approach to geophysical fluid transport”, Emilio Hernandez-Garcia, Dec 12.

“Active cluster crystals”, Cristóbal López, Dec 12.

Awards:

Stipendium Hungaricum Scholarship

The Second Rank Academic Scholarship Of China For PhD, 2018.9 - 2019.6

Research Allowance of CCNU For PhD, 2018.10 - 2018.11