Localisation in lattice gauge theory
Semester report: Semester II.

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1 Introduction

The low-lying Dirac modes become localised at the finite-temperature transition in QCD and in other gauge theories, suggesting a general connection between their localisation and deconfinement. $\mathbb{Z}_2$ gauge theory in 2+1 dimensions is the simplest model where this connection can be tested. Last semester I investigated the localisation properties of this gauge model. According to the sea/island picture, the Dirac operator favors sites where the phase of Polyakov loops is close to $\pm\pi$. In $\mathbb{Z}_2$ gauge theory the phase of Polyakov loops can be either 0 or π, thus due to the sea/island picture it is clear where one expects localisation of Dirac modes. On the other hand, in $\mathbb{Z}_3$ gauge theory the Polyakov loop phase cannot get closer to $\pm\pi$ than $\pm\frac{2}{3}\pi$ (among $\mathbb{Z}_N$ models the minimal distance from the phase $\pm\pi$ is the largest in this gauge theory). It is then worth checking whether localisation of the low Dirac modes is present in the deconfined phase of this model, working for simplicity in 2+1 dimensions. Since $\mathbb{Z}_3$ is also the centre of the gauge group of QCD, this may be useful to better understand the possible role played by centre symmetry in the localisation of Dirac modes in that case.

2 Research

To study how localisation properties change from the confined to the deconfined phase in the 2+1 dimensional $\mathbb{Z}_3$ gauge theory at finite temperature, one has first to determine the critical temperature, which has not been done yet in the literature. To determine the critical temperature more efficiently, one can exploit the dual model, which is the three-colour Potts model. Such a model can be numerically simulated using a cluster algorithm, thus enormously improving over standard algorithms in the vicinity of the phase transition, which is expected to be a second-order phase transition in the universality class of the 2-dimensional three-colour Potts model.

In general, in $\mathbb{Z}_n$ gauge theories the link variables are

$$U_l = e^{2\pi i\theta_l/n} \tag{1}$$

where $\theta_l = 0, 1, ..., n-1$. The action is

$$S = \sum_p \left(1 - \cos\left(\frac{2\pi\theta_p}{n}\right)\right), \tag{2}$$

where we sum over all plaquettes. By Fourier-expanding each plaquette and summing over the link configurations, one can recast the partition function in terms of spin-like variables living on the sites of the dual lattice, where each link pierces the middle of a plaquette of the original lattice. The resulting model, the *clock model*, is a generalisation of the Ising model. In clock model the energy takes the following form:

$$E = -\beta^* \sum \cos(2\pi(\theta_i - \theta_j)/n), \quad \theta_i = 0, 1, ..., n-1, \tag{3}$$

where β^* is the coupling constant of the dual model. The connection between the dual models is:

$$\beta^* = \frac{2}{3}\log\left(\frac{e^\beta - 2e^{-\beta/2}}{e^\beta - e^{\beta/2}}\right). \tag{4}$$

Another generalization of the Ising model is the Potts model. The energy in the Potts model is:

$$E = -\beta^{**} \sum_{<ij>} \delta_{s_i s_j}, \quad s_i = 0, 1, 2, .., n-1, \tag{5}$$

where summation runs over all neighbouring sites, s_i and s_j are the spin variables and β^{**} is the coupling. One can prove that the 3-state Potts model and clock model are equivalent with

$$\beta^* = \frac{3}{2}\beta^{**}. \tag{6}$$

Cluster algorithms are available for the Potts model, and can be used to determine the critical β^{**}, and so the critical beta of the $\mathbb{Z}_3$ gauge model.
The numerical code is ready for the study of the critical temperature, which will begin soon. After this part of the programme is completed, I plan to study the localisation properties of Dirac eigenmodes in the $\mathbb{Z}_3$ gauge model.

3 Publication and conference

During this semester me and my supervisor have completed a manuscript on the localisation properties of eigenmodes of the staggered operator in $\mathbb{Z}_2$ gauge theory, arXiv: 2104.03779, currently submitted to Physical Review D.

On July 26-30 I will take part in the International Symposium on Lattice Field Theory, where I will give a talk about localisation in $\mathbb{Z}_2$ gauge theory.

4 Studies

This semester I attended *Solitons and instantons I.*, *Beyond the standard model* and *Quantum Information Theory* .

5 Teaching

This semester I took part in teaching the course *Klasszikus fizika laboratórium* for second year B.Sc. students as a lab assistant (4 hours a week).