Semester Report

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PhD Program: Statistical physics

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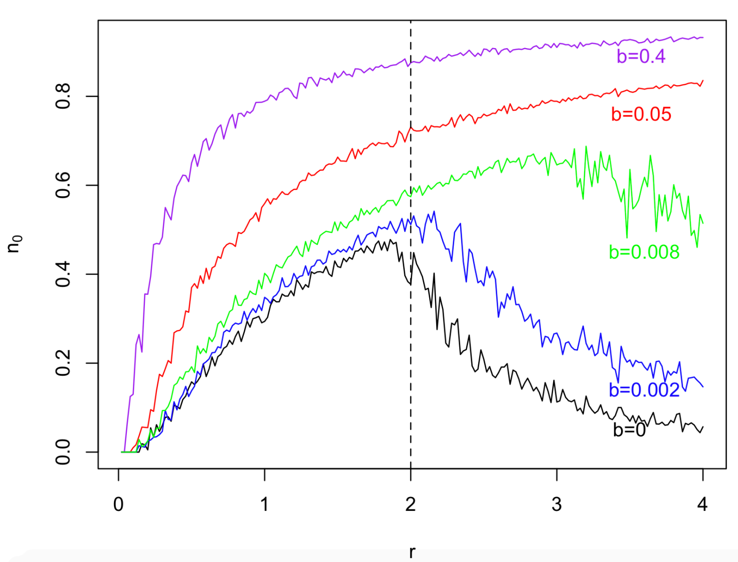
Introduction:

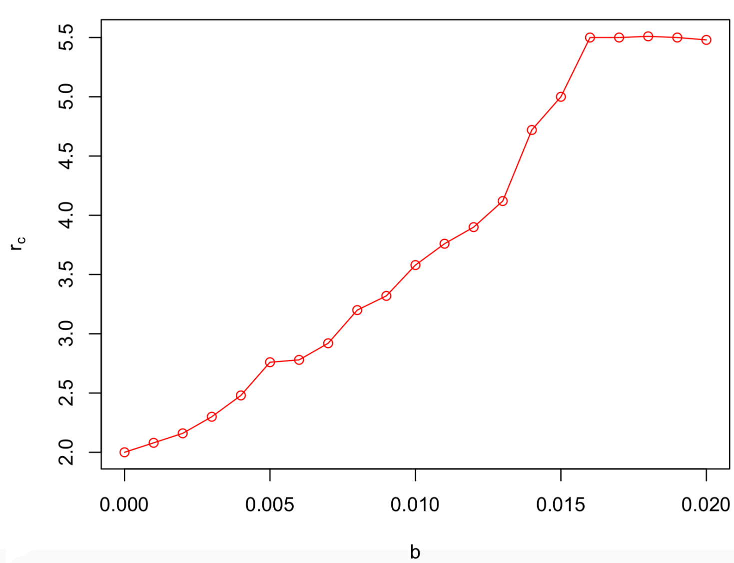
The optimization of variance supplemented by an asymmetric L_1 regularizer is a representative example of quadratic optimization and can be carried out by replica method analytically. The recent research by *I. Kondor et al. 2019* studied the behaviors of optimization under L_1 constraint for density of zero weights n_0, regularization parameter η, distribution of weight $p(w)$ and some order parameters, which show that these quantities are independent on the ratio r of the portfolio's dimension N to the sample size T, and on the strength of the regularizer. From their results, optimization under a spacial case of L_1 constraint, no short-selling($\eta_1 = 0, \eta_2 = \infty$), has a phase transition taking place at $r = 2$ (also see *I. Kondor et al. 2017*). However, the covariance matrix σ in risk function of optimization in those works are all regarded as an identity matrix for more convenient to find the result of quantities. By changing the structure of covariance matrix to a symmetric non-diagonal matrix, the critical value is no longer $r_c = 2$, but shift with the value of non-diagonal elements in covariance matrix, which I found during this semester.

Description of research work carried out in current semester:

By analytical method of replica trick, the ratio $r = N/T$ can not be large than 2 because of the limitation of Lagrange multiplier in building the objective function, so the works here are all in simulation way by *R* language with the help of "*quadprog*" package. No short-selling($\eta_1 = 0, \eta_2 = \infty$) optimization is considered as a simple and special case of L_1 constraint optimization because they have the same behavior of quantities in most of the time. To find accurate results, N and T must take large values, especially for generating the data of zero weight rate n_0. If N and T are small, the probability of zero weights would have a big deviation by simulation with the same N and T. Instead of identity matrix, here we introduce the structure of covariance matrix in risk function as the all diagonal elements equal to 1 and set other elements as a variable b where $b \leq 1$. To test the different value of b, the relations between the structure of covariance matrix and those quantities of optimization are displayed clearly.

Elimination of zero weight density n_0 with increasing b from $b = 0$ to 0.4 is showed in the left figure below. With $T = 100$ fixed and 100 times averaging, r changes only with different values of N. It is clearly that n_0 increases to 0.5 at critical value $r_c = 2$ when $b = 0$, which is consistent with the result of *I. Kondor et al. 2019*. However, the curve starts to decrease above $r = 2$ and fluctuation become large, so there is is a peak value of n_0 at r_c. When b is no longer zero and

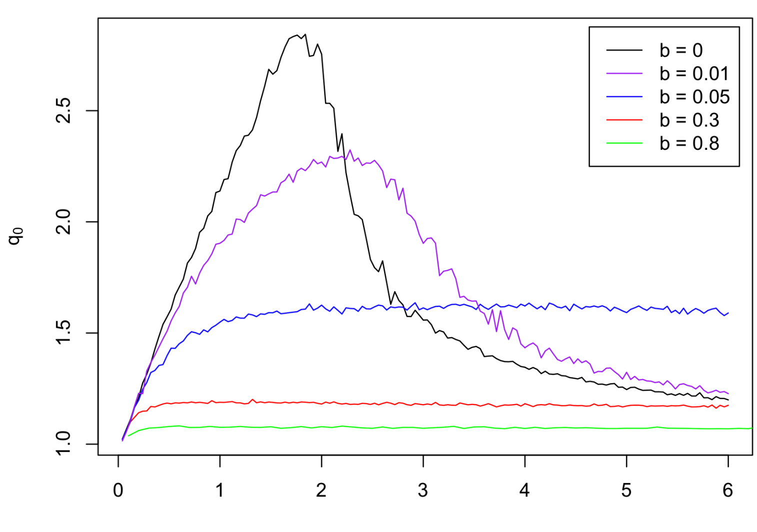


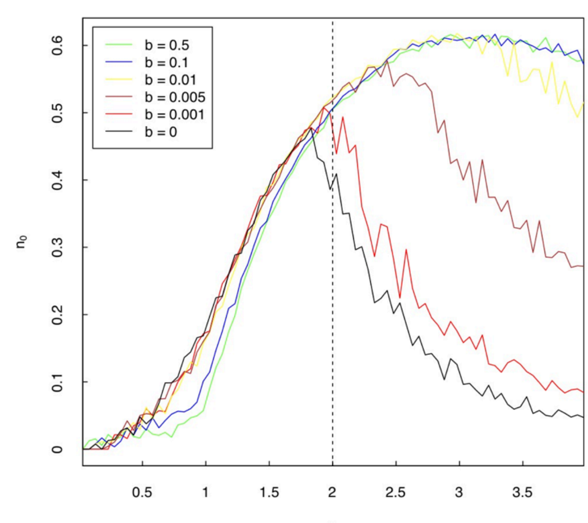


increases with small values, r_c also shifts with small steps. The peak value of n_0 disappears when b is large enough and then the curve of n_0 becomes an increasing function of r to $n_0 = 1$. In the right figure above, r_c moves from 2 to 5.5 where b grows from 0 to 0.17, then the peak value in the curve of n_0 disappears and the density of zero weights becomes steady around $n_0 = 0.8$ which goes to 1 as $b = 1$ at last.

The order parameter q_0 , a measure of the estimation error, also has a similar behavior with the change of b (the left figure below). The movement of critical ratio r_c related to b is still tested by simulation which may be the same behavior as n_0. The similar behavior in optimization under L_1 constraint is generated out either for $\eta = 0.1$(the right figure below), which means this kind of behavior might be more universal in optimization models.

The further works are a bit more. The first part is to test wether each important quantities of no-short model has the similar behavior related with b, i.e., the structure of covariance matrix, and evaluate accurate critical values. Secondly, we know that optimization under L_1 constraint maybe





has the similar behavior of those quantities, but wether the quantities have the same changes related with d under different values of regularization parameter η? That also should be checked. And the last, how about other quadratic models? Some models would be tested around to prove our guess.

Studies in current semester:

ELTE courses:

FIZ/3/054E	Universality classes in non-equilibrium system	(Dr. Ódor Géza)
FIZ/3/085	Data Exploration and Visualisation	(Dr. Dobos László, Dr. Visontai Dávid)
FIZ/2/092E	Quantumelectrodynamics	(Dr. Nógrádi Dániel)
FIZ/3/004E	Fractal growth	(Dr. Palla Gergely, Dr. Pollner Péter)

Conferences in current semester:

Statistical Physical seminar at ELTE Institute of Physics:

"Thermodynamics of the O(4) model from the Phi-derivable approximation", Zsolt Szép, Feb 20.

"Locality, short-time behavior and reconstruction of Hamiltonians of continuous time quantum walks", Balázs Szigeti, Feb 27.

"The role of the surrogate time series in exoplanetary dynamics", Tamás Kovács, Jun 5.

Awards:

Stipendium Hungaricum Scholarship

The Second Rank Academic Scholarship Of China For PhD, 2018.9 - 2019.6

Research Allowance of CCNU For PhD, 2019.03 - 2019.05

Research Allowance of Institute of Particle Physics of CCNU For PhD, For Spring Semester